FAILURE ANALYSIS OF CLAMPED RECTANGULAR PLATES UNDER IDEALIZED ICE-LOADS

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Submitted to the Department of Naval Architecture and Marine Engineering in partial fulfillment of the requirements for the degrees of Naval Engineer and Master of Science in Naval Architecture.

ABSTRACT

The objective of this work was to determine the expected deflections prior to and at the rupture load for a panel of hull plating of an ice-strengthened ship. An idealized mathematical model of a pinnacle of ice being forced against the plating was used as the assumed loading mechanism. The panel of hull plating was modeled as a clamped rectangular plate. The idealized ice-loads were defined as a four sided pyramid of ice which when forced against the center of a rectangular plate crushed at a given pressure. Thus the size of the load bearing surface continually increased as the applied force was increased.

A method of analysis was established which provides a step-by-step solution of the large deflections of a plate and a means of determining the minimum expected failure load. The analysis is begun at the point where the load on the plate from the pyramid of ice is large enough to cause plastic collapse by a three hinge mechanism. A solution method for the deflection of a uniformly loaded membrane forms the basis for the calculations of the plate deflections. Then the energy absorbed by the plate at the point of expected failure is calculated from an equation that depends on the total strain at the point of failure.

The possibility of failure of the plate at any given load and deflection may be determined from the failure criterion. This criterion was established by forming a relation between a parameter representing the geometric properties and a parameter representing the material properties of the plate. The relation was formed from published data on plates that had been tested to failure. The calculated failure parameters for these plates were used to form a linear equation which correlates all the experimental failure loads. The criterion was then checked by comparing the results given in other experiments on the large deflections of plates.

The method of analysis was then applied to plates similar to the hull plating of ice-strengthened ships. The results showed that the method of analysis can be used with a reasonable degree of accuracy to calculate the large deflections of a plate subject to a lateral load and to predict the load and deflection at which failure may be expected.

Thesis Supervisor: J. Harvey Evans
Title: Professor of Naval Architecture

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The foremost acknowledgment is for the patience and understanding of my wife and family.

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NOMENCLATURE

F, F_n, F_o	--(lbs)	The force behind the pyramid of ice
P_c	--(psi)	Ice-crushing pressure
q	--(psi)	Any uniformly distributed load over a previously defined bearing area
M_x	--(in-lb)	The bending moment about the longitudinal axis
M_o	--(in-lb)	The bending moment at the plastic collapse load.
σ_o	--(psi)	The yield stress for a material
σ_u	--(psi)	The ultimate stress of a material
σ_p	--(psi)	Equivalent yield stress of material in an elemental strip of plate
σ_s	--(psi)	In-plane membrane stress
σ_t	--(psi)	Total in-plane stress due to the membrane force and bending moment
S, S_o, S_i	--(lb/in)	In-plane membrane force per unit length of plate edge
U	--$\left(\frac{\text{in-lb}}{\text{in}^3}\right)$	Energy per unit volume absorbed by the plate
R	--$\left(\frac{\text{in-lb}}{\text{in}^3}\right)$	Total restraint energy per unit volume of a material in simple tension
ϵ_u	--(in/in)	The ultimate strain of a material in simple tension corresponding to σ_u
ϵ_t	--(in/in)	The total strain at a point due to bending and stretching
ϵ_x	--(in/in)	The strain in the x-direction at a given point in the plate
ϵ_y	--(in/in)	The strain in the y-direction at a given point in the plate
ϵ_z	--(in/in)	The in-plane strain of the outer fibers of the plate

Z, Z_i, Z_{i+1}	--(in)	Partial center deflection as determined by membrane formula
w	--(in)	Total plate deflection at any point (x,y)
W, W_i, W_{i+1}	--(in)	Total center deflection for a given load
ψ	(radians)	Angle between horizontal and midplane of plate at a given deflection
θ	(radians)	Angle of rotation due to bending of a plane cross-section through the thickness of the plate
$\delta = W/A$		A non-dimensional failure parameter for plate geometry
$\phi = U/R$		A non-dimensional failure parameter for plate material properties
k		Coefficient within the membrane formula that is dependent on x and y
k_d		Coefficient within the derivative of Z that is dependent on x and y

INTRODUCTION

Discussion of Problem

The solutions to two interrelated problems are sought in this analysis. The failure of clamped rectangular plates may be separated from ice-loading of plates, but the two studies are brought together here for the consideration of a special problem in ship's structures. The full analysis of hull plating subjected to the various loading conditions associated with navigation through areas where ice is present brings the two topics together. The first problem is to determine the relation between the hull plating and the ice that may be forced against the plating. The second problem is encountered as the force driving the ice against the ship is increased and the hull plating deforms until plate failure or rupture occurs. The total process may be considered continuous, starting with the initial contact with the ice, subsequent ice crushing and plate deflection, and finally, extreme plastic deformation of the plate and failure.

The need for this analysis is generated more from the design of merchant ships which are to operate in ice filled waters than from icebreaker design. Icebreakers are generally very conservatively designed because they must be able to withstand ice-loadings of almost any magnitude during their lifetime. On the other hand, merchant ships, even those with icebreaker bows, have to balance their design between profit and repair costs. Their carrying capacities are greatly affected by the addition of thicker steel plates along the ice-belt. Furthermore, if there is a chance that the ship could lose part of its cargo through

a ruptured plate, the economics of the situation increases. Even more important though, is the rising concern over the possibility of oil pollution of the arctic seas. In order to prevent oil spills, it has been proposed that a tanker be designed with outer buffer tanks filled with water. The outer hull would be of thinner plate than the usual single hull ship. The separation of the two hulls would be based on the maximum deflection that could be expected from the outer plating when it failed under ice-loads.

Idealization of Ice-Loads

The ice-loads to be considered will be idealized with regards to their geometry and point of application. The present design practices are based on static loading of the hull plates. The model loads used here will be considered continuous but applied in incremental static steps. Icebreaker and ice-belt plate scantlings are usually obtained by using a uniformly distributed pressure equal to the ice crushing pressure. Since the actual ice crushing pressure is not constant and dependent upon geographical location, age, salinity and many other factors, it was desired to try to cover a wide range of possible loading mechanisms. The driving force behind the plate/ice interaction must also be considered. It was for these reasons that a pyramid shaped pinnacle of ice was chosen as the idealized ice-loading mechanism.

Type of Analysis

Plate failure analysis lies between the theory of plasticity and a handful of experimental tests that have carried applied loads up

to plate rupture. Theoretically, it would be desirable to use the theory of plasticity to analyze the plate deflection and failure. This was considered, but after reading other works in the field and considering the accuracy of the possible results, it was decided to use a more empirical approach. The final analysis will be a combination of plastic analysis and engineering judgment. This combination of approaches to the problem allows a study of the plate/ice interaction within a limited time period with sufficient accuracy of the results.

Load/Deflection Relations

Most of the experimental work on rectangular plates with large deformation has been done under uniformly distributed loads. To obtain any data on concentrated loads, it was necessary to go back to a report of experiments conducted in 1937. The plate deflections and stresses were kept in the elastic range in these tests. The results did provide a good starting point for the shapes of the plates during plastic deformation under concentrated and partial loads. Some of the results and observations on circular plates under concentrated loads were borrowed for the analysis.

It was necessary to consider the concentrated or partial loads in order to present the complete model of plate/ice interaction. The crushing process of ice in contact with the ship's hull will begin as a concentrated load. The ultimate strength of sea ice in compression is usually low enough in relation to the design pressures of hull plating such that the ice-loads are uniformly distributed before permanent deformation of the plate begins. However, since the ice crushing process will

require some partial loading, large plate deformations may occur under smaller, but more concentrated loads. Therefore, partial loading of the plate will be analyzed in order to consider all possible extremes for the failure analysis of the plate.

Failure Analysis

Experimental results of rectangular plate failures are usually an added column of data to the experimenter's primary reason for testing. The beginning of investigations into large plastic deformation and failure is often attributed to Bach. His experiments conducted in 1908 formed the basis for Hovgaard's 20% permanent deflection rule.[1] These tests, like most of the data obtained for rectangular plate failure, were for uniformly distributed loads. Often the comment that the plate failed at a given pressure is the only amount of data given. Greenspon, [T4] in evaluating experiments by J. W. Day, [E2,E3] attempted to correlate the failure data so that plate rupture under a uniformly distributed load could be predicted. His recommendation was to establish a limiting maximum deflection to span ratio. Using a statistical approach, it could be stated that a uniform lateral load giving a deflection to span ratio greater than 0.10 could be considered approaching the plate failure load. The only other author who discussed plate failure to any extent was Loeser. [E1]

Most of the experiments as reported by the authors listed in the bibliography under Plate Deflection-Experimental were helpful in establishing a failure criterion. It was decided early in the analysis that an energy relation would be used. This enabled correlation of the

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1 See reference T5, pp. 120-125.

failure data that was available. Not all of the data was useful, since all material properties must be known in order to properly correlate the experiments. Since Day and Loeser included all material properties in their reports, the results of their experiments were relied on most heavily.

Outline of Procedure

The procedure of investigation that follows will continue in a similar format as used in the introduction. First, the model of the ice-loads will be formed. Then, the plate deflection/load relations will be established. At each increment of loading or deformation of the plate, the amount of energy absorbed by the plate material should be calculated. The relations for this segment of the analysis will depend on the load/deflection calculations and will serve to relate them to the failure criterion. The final step in the analysis will be the establishment of the failure criterion. This will complete the model of the ice/plate interaction.

Ordering of the Bibliography

It has been previously mentioned that this analysis falls between the basic theory of plasticity and empirical relations. Many of the assumptions and choices of methods are based on the experiments and observations of other authors. Each phase of the analysis can be performed by other techniques. However, within the time limits of the investigation, all cases were carefully studied so as to make use of the best method available. For these reasons, it was felt that the list of

references should reflect the degree of influence of each work upon the final analysis. Therefore, the references within each subject area are listed in approximate descending order of influence or use.

PROCEDURE FOR CALCULATING ICE/PLATE INTERACTION

Initial Assumptions for Ice-Loads

A four sided pyramid of ice as shown in Figure 1 will be the loading mechanism. The manner of loading will be such that it may be considered to be static at all times.

The included angle of the pyramid frustum will not effect the load bearing area of the ice on the plate. Also it is small enough to cause only the crushing portion of the ice to make contact. That is, the sloped sides of the cone will not come in contact with the plate at any time.

The load bearing surface will conform to the curved shape of the plate as if the plate in the region of the load is subjected to a uniformly distributed load. Thus, load edge stress concentrations will be neglected. Also, at no time will the area of load bearing surface be less than twice the thickness. Therefore, shear forces will be neglected.

The load bearing area will be a square of side equal to C. If C equals the width of the plate, the load bearing area will be set equal to the plate area. Also, at this point, the loading pressure will be shifted from a constant ice crushing pressure to a pressure equal to the total load divided by the plate area.

Initial Assumptions for Rectangular Plates

The plate material will be isotropic and homogeneous. All four sides of the plate will have a clamped edge condition for support as shown in Figure 2 and only in-plane stresses are to be considered.

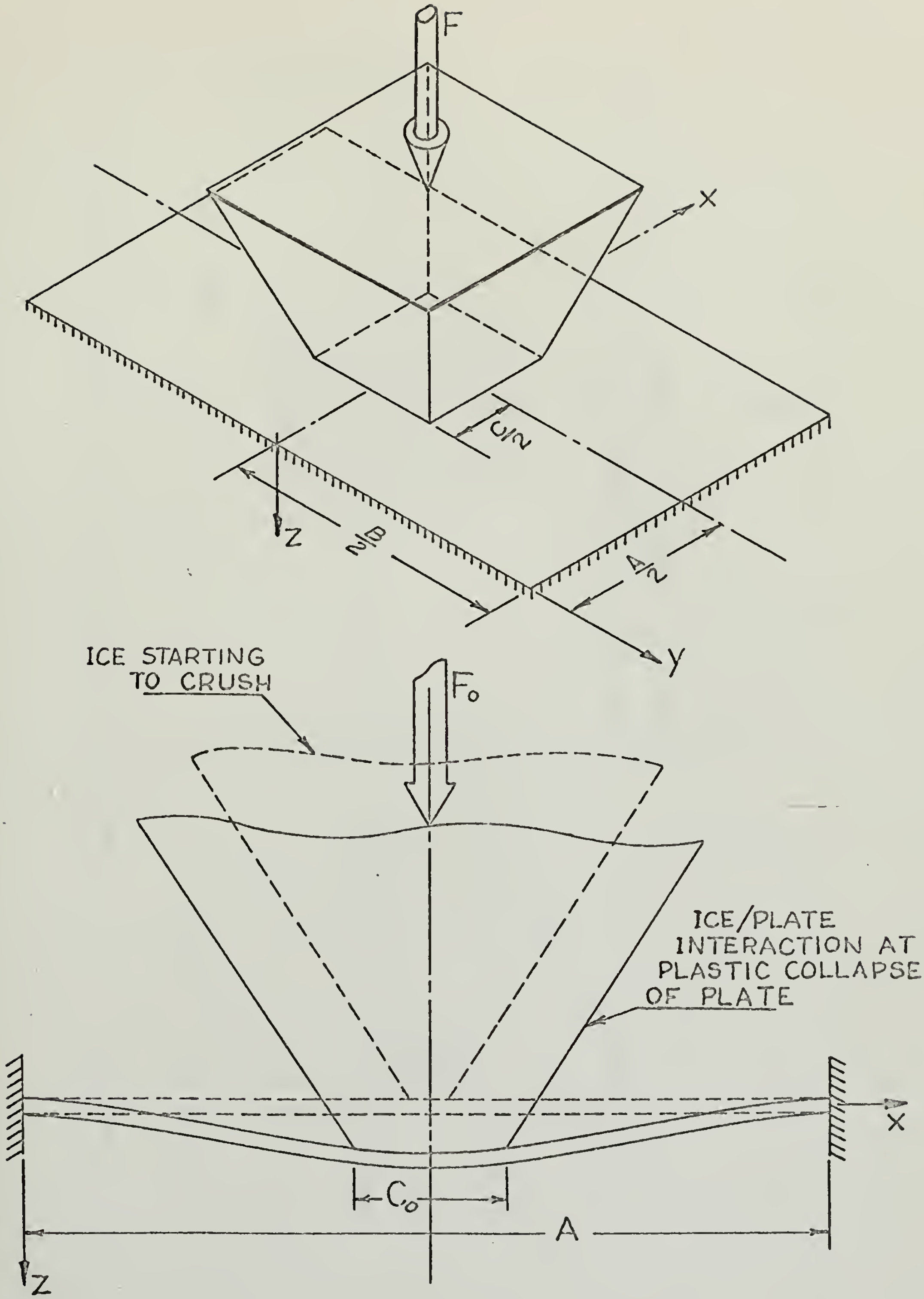

Figure 1. Idealized Ice-Load

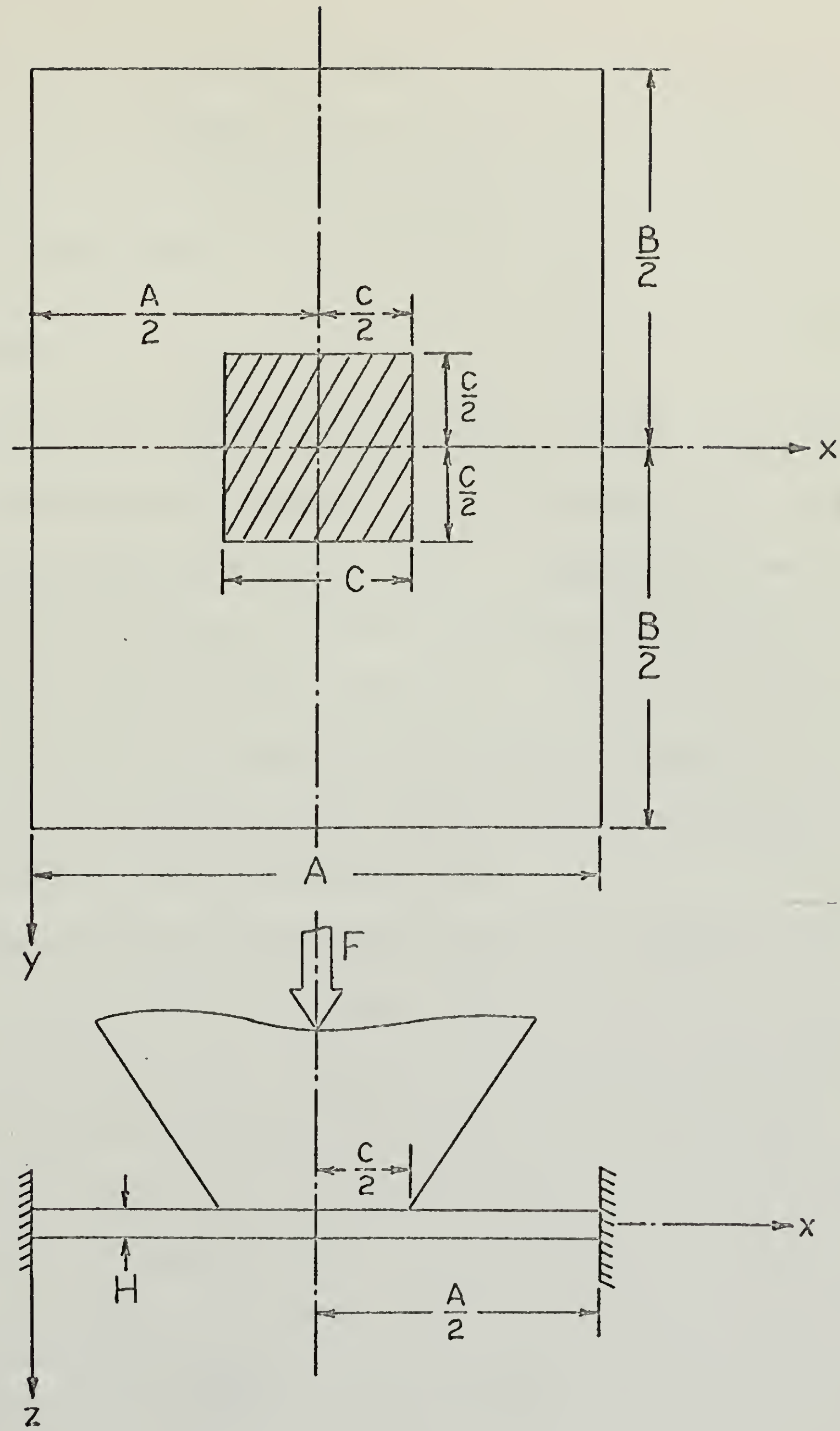

Figure 2. Rectangular Plate Dimensions and Load Position

Only deflections after the plastic collapse load is reached will be considered. A rigid, perfectly plastic material is assumed for the collapse mechanism and a biaxial strain condition will be assumed at the mid-point of the plate.

Model of Ice-Loads

The idealization of an ice-load will be a four sided pyramid of ice. The actual applied load will be in the form of a force acting on the base of the pyramid. The force will be transmitted to the rectangular plate in the form of a uniformly distributed load acting over an area centered at the mid-point of the plate. The value of the loading pressure will be the ice crushing pressure except during initial loading and after the bearing surface is equal to the plate surface area. These exceptions will be discussed later.

As the driving force increases, the bearing surface of the pyramid will increase as the ice crushes. Thus, if the ice crushing pressure is considered constant throughout the loading process, the bearing surface area will increase directly with the driving force.

Since the area, C^2, at any time is equal to the driving force, F, divided by the ice crushing pressure, P_c, the length of the side of the bearing area will be:

$$C = \sqrt{\frac{F}{P_c}} \tag{1}$$

This relation will hold after the pyramid first starts to crush until the plate is completely covered by the bearing surface. The loading prior to the initiation of ice crushing will not be considered, since

the plate deflection will be in the elastic range.

Once the bearing surface equals the area of the plate, the applied load, F, will remain constant unless P_c is allowed to increase. Thus it will be assumed that ice crushing ceases and P_c will increase with any increase in F as:

$$P_c = \frac{F}{AxB} \tag{2}$$

Plastic Collapse Load

The collapse mechanism for the plastic analysis is the general roof shaped deflection surface. For the transverse cross section a three hinge collapse mechanism is used. For any loading, it was assumed that the center plastic hinge forms either simultaneously with the two edge hinges or at some increased load beyond the formation of the edge hinges.

Since the analysis is for large deflections of plates the deflection at the plastic collapse load will be the first to be considered. Unfortunately, there is no easy solution for the deflection at this load. The problem is further complicated when partial loads are considered. Finite displacement relations from the theory of plasticity and a membrane solution were considered to be the easiest methods for finding the deflections. A similar degree of accuracy may be obtained from either method. The membrane solution has been shown to provide good representation of the plate deformation for loads well beyond the collapse load. Therefore, it was decided to use the membrane solution throughout the analysis.

As the pyramid of ice is forced against the plate, the pressure acting over the initial bearing surface will rise until the ice crushing pressure is reached. During this time, the plate will behave elastically. From the time P_c is first reached until plastic collapse of the plate, the area of the bearing surface will increase. There will be associated with the collapse load F_o, a constant P_c, and an area C_o^2. In order to use the membrane solution to find the deflection at collapse, the value of C_o must be known. Actually it is easier to use the non-dimensional form C/A.

The moment M_x at the center of the plate was found using an elastic solution for clamped plates under partial loads. $M_x = \beta' P$ where P is the total load and the load coefficient β' is given in Figure A1 for values of B/A and C/A. Then by setting $M_x = M_o = \sigma_o H^2/4$, the yield moment for a rigid, perfectly plastic material, the bearing surface area can be related to the yield stress and plate thickness. Thus, when $P = P_c C^2$ and both sides of the equation are divided by the width of the plate, A, the following non-dimensional equation is formed:[2]

$$\frac{\sigma_o}{P_c} = 4\ \beta' \left[\left(\frac{A}{H}\right) \left(\frac{C}{A}\right)\right]^2 \tag{3}$$

From equation (3) the bearing surface area at the collapse load may be determined for any plate. Various solutions to the equation are given in Figures A3-A5 for given values of B/A and A/H.

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[2] This is equation (A7) of Appendix A.

Plate Deflection Calculations

Once the size is determined for the load bearing surface at the collapse load, it is necessary to calculate the deflection of the plate at this load. As discussed previously, a membrane solution will be used for the deflection calculations. The membrane formula,[3]

$$Z = \frac{4qA^2k}{\pi^3 S} \tag{4}$$

in general only holds for a uniformly distributed pressure over the entire plate. This meant that some geometric relationships were required in order to use the membrane formula with partially loaded plates.[4] It was assumed that the plate in the region of the partial load behaves as a membrane after plastic collapse was reached. Then the membrane or in-plane forces along the edge of the loaded area are carried through to the edge of the plate. This implies that the slope of the deflection profile at the edge of the loaded area is the same as the slope at the edge of the plate. Thus the conical shape is represented for the concentrated loads and the full membrane shape is used when the load is uniformly distributed over the plate. It is believed that this provides a good representation of the true deflection. The deflections for the concentrated loads may be less than they should be, but the accuracy should improve as the load bearing area increases.

The total deflection calculations consist of three basic steps. First, the deflection Z for the loaded portion of the plate is found from the membrane formula. For the width of the applied load C

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[3] See Appendix B.

[4] See Figure 3.

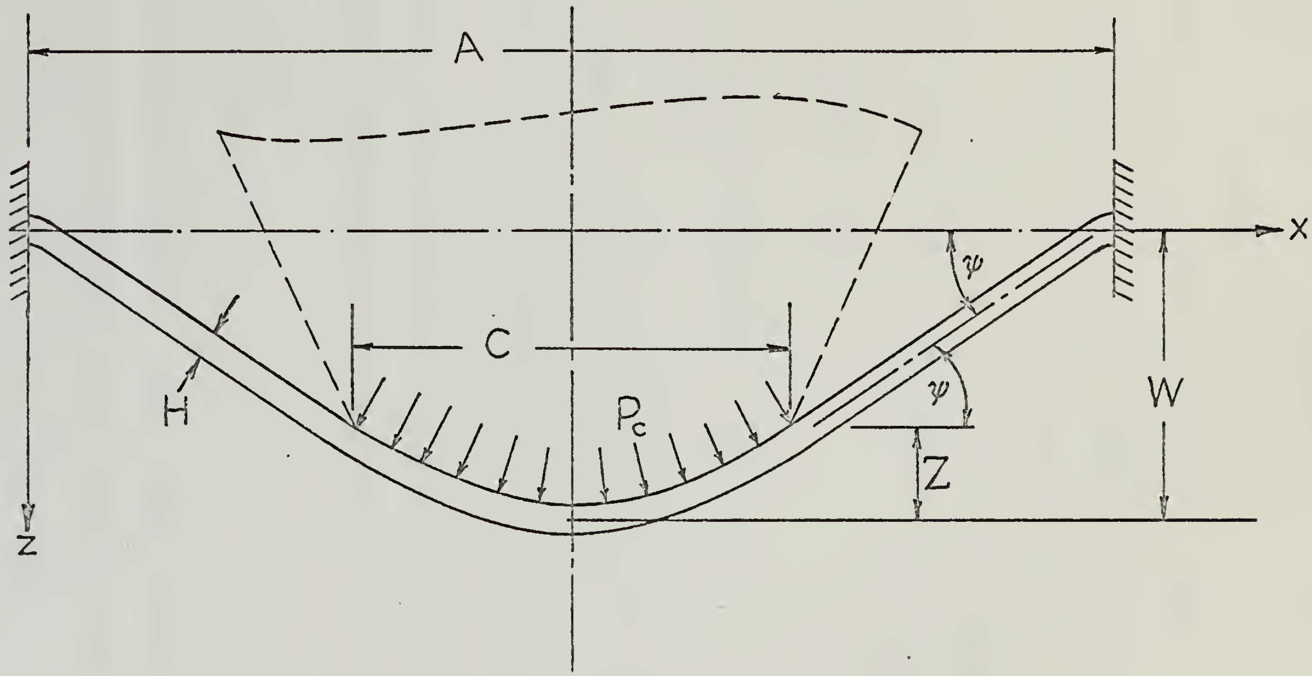

Figure 3. Plate Deflection and Partial Load Relationships

and the pressure P_c, Figure 3 shows the relation of Z to the total deflection. The second step is to determine the slope of the plate at the edge of the loaded area. The derivative of the deflection profile provides the slope at the point $x = (A \pm C)/2$. Finally, the total deflection at the mid-point of the plate is found by combining the membrane solution with the assumed slope at the plate edge. A simple geometric relation will determine the deflection at the edge of the loaded area, and the membrane solution may be added to it to give the total deflection. The central deflection W is given by:

$$W = Z + \frac{1}{2}(A-C)\tan\Psi \quad , \tag{5}$$

where the membrane formula

$$Z = \frac{4\,C^2\,k\,P_c}{\pi^3\,S} \tag{6}$$

is the deflection of the loaded portion,

$$\text{and } \tan\Psi = \frac{d\,Z}{dx} = \frac{P_c\,C\,k_d}{2S} \quad \text{at } x = \frac{A \pm C}{2}\,. \tag{7}$$

The constants k and k_d are the factors that depend on x. The values for these constants are included in Appendix B for given plate aspect ratios.[5]

Prior to discussion of the procedure for load/deflection analysis, the membrane solution for plate deflection must be understood. The actual formulation of the deflection relation is given in Appendix B. There are two dependent variables in the relation. The membrane force, S, which acts along the edge of the loaded area depends on the total deflection of the membrane, W, which in turn, depends on S. The

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5 See Figure B1.

usual procedure for obtaining the value of S is to assume a membrane deflection and shape. Then a strain relation is used to obtain the principal strain in the plane of the plate for the assumed deflection. The usual assumed deflection shapes are cylindrical or parabolic.

In order to model the ice loading condition, the equation for the shape of a partially loaded, simply supported plate was used. A biaxial strain relation which depends on the B/A ratio, the size of the bearing surface given by the C/A ratio, as well as the maximum deflection of the plate was established for the mid-point of the plate.[6] This made it possible to obtain the membrane stress from the stress-strain curve for the plate material. Thus, for a given deflection, the membrane force per unit length is equal to the membrane stress times the plate thickness.

If the membrane formula (equation 4) is used alone, the applied load, q, could be found that would produce an assumed deflection. Since, for the model of ice-loads, the loading pressure is assumed constant, an iterative procedure must be used to obtain the proper correlation between Z and S for a given constant load P_c.

For the calculations involving the deflection of a partially loaded plate, the total deflection, W, is used instead of Z to determine the value of the membrane force, S. This appears to provide a more realistic situation, because the strain relation as derived in Appendix D depends on both the plate shape and deflection.

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[6] See Appendix D.

Load/Deflection Relations

The procedure for obtaining the deflection, W, for each value of the total load, F, will be iterative. The beginning will be based on the plastic collapse deflection. After the value of $(C/A)_o$ is found from the plastic collapse relation, the membrane solution will be used to find the deflection at the plastic collapse load, F_o. The first approximation for the deflection is found by assuming that the membrane stress is equal to the yield stress. Then for the value found for C_o from $(C/A)_o$ and the ice crushing pressure, P_c, the first values of Z, dZ/dx and W (equations 6, 7 and 5) can be calculated. This is the starting point of the iteration for the proper value of W_o for the given load F_o. The successive solutions of W will follow the same format as outlined below for the general solution.

For any given total load F_n, there will be a corresponding bearing surface area represented by the length of one side C_n. The problem is to find the deflection W_i that corresponds to F_n and C_n.

Let $F_n = F_o + \Delta F$ where the subscript n designates a certain value of static load, and ΔF is the incremental value of the total load. The subscript o refers to the previous value of applied load. For example, F_o will initially represent the collapse load. Then

$$C_n = \sqrt{\frac{Fn}{P_c}} \qquad (8)$$

will provide the necessary load bearing area to determine the value of W_i.

The previous value for membrane force will be used to obtain

a membrane deflection for C_n. That is,

$$Z_i = \frac{4\ C_n^2\ \ k\ P_c}{\pi^3\ S} \tag{9}$$

and it follows that

$$\left(\frac{dZ}{dx}\right)_i = \frac{P_c\ C_n\ kd}{2S} \tag{10}$$

and

$$W_i = Z_i + \frac{1}{2}\ (A-C_n) \left(\frac{dZ}{dx}\right)_i \tag{11}$$

The new values W_i and C_n are used to enter the biaxial strain relations and obtain a new value for the membrane force S_i. The next step is to return to equations (9), (10), and (11) to calculate Z_{i+1}, $(dZ/dx)_{i+1}$, and W_{i+1}. The subscript i+1 is used to distinguish the new values of deflection from the membrane force. This notation indicates that the deflection calculation is made with the membrane force found from the previous deflection, W_i. After each new value of W_i is found, its corresponding membrane force S_i is determined from the strain relations and compared with the previous value of S. Thus, the calculation will be concluded when the two values of the membrane force compare favorably.

The above procedure is repeated for each new value of F_n. When the load is reached which produces a load bearing area equal to the area of the plate, that is when C = A, then the loading pressure P_c will cease to be constant and increase with the total load. For all subsequent increases in load C/A = 1.0 and $P_c = F_n/(A \times B)$. This assumption of continuous loading on the plate is just an extension of the model of the ice loading beyond the point where the pyramid of ice has been fully crushed.

Energy Absorbed Relations

Once the load/deflection relation is found, the energy absorbed by the plate should be determined at each value of F in order to evaluate the possibility of plate failure. The energy of deflection will be the greatest at the edge of the plate except for cases of very small load bearing surfaces. For the plate thicknesses usually considered for ships, the load bearing areas will be large enough to rule out the shear effects of the concentrated loads.

Since the plate failure can be expected at the mid-point of the long side of the plate, the energy absorbed at this point should be greater than that at any other point. There will be two sources of strain energy at the edge of the plate. The initial source will be from the bending moment. Then as the membrane forces increase, the portion of the total strain due to the bending moment will decrease. The combination of the strain due to bending and stretching will give the total strain at the expected point of failure.

Using the assumption that plane cross-sections remain plane during the bending and stretching, the strain distribution across the plate thickness will be as shown in Figure 4. The rotation of the cross-section is due to the plate bending, and the location of the neutral axis away from the mid-plane is determined by the size of the membrane stress. The influence of the membrane stress is equivalent to lowering the neutral axis from the geometric center to a new point z_1. This shift in axis of rotation of the cross-section depends on the

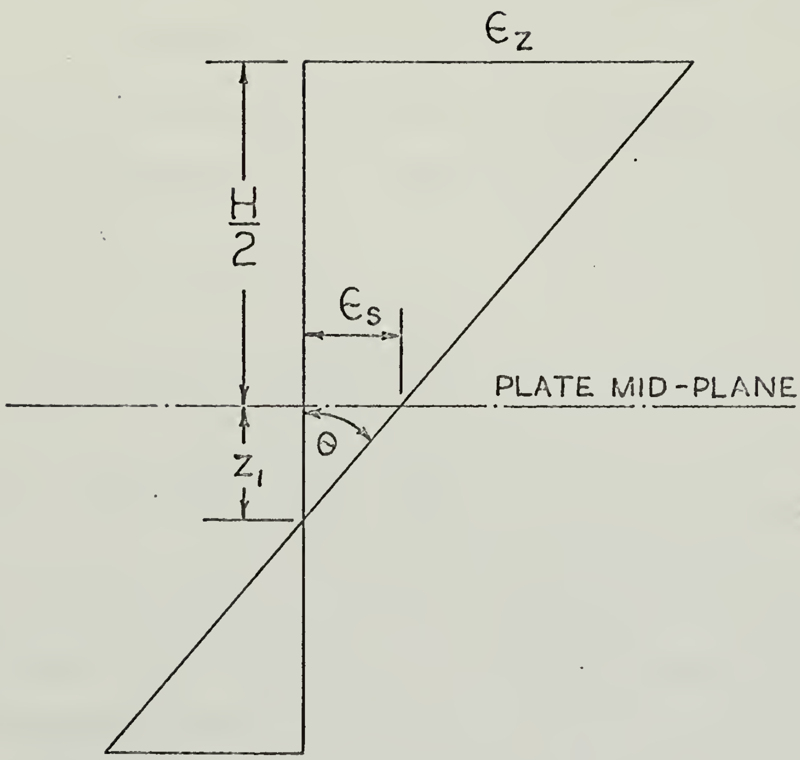

Figure 4. Strain Distribution for Combined Bending and Membrane Forces

ratio of the membrane stress to the equivalent yield stress;[7]

$$z_1 = \frac{H}{2}\frac{\sigma_s}{\sigma_p} \tag{12}$$

$$\text{where } \sigma_p = \frac{\sigma_o}{\sqrt{1-\nu-\nu^2}} = 1.156\,\sigma_o \text{ for } \nu = 0.5 \tag{13}$$

This relation will hold true for values of σ_s greater than σ_p since at that point the neutral axis will lie "outside" of the material and all strains across the thickness will be in tension.

The strain of the outermost fiber at the mid-point of the long side will be represented by ϵ_z. From Figure 4,

$$\epsilon_z = \epsilon_s + \frac{H}{2}\tan\theta \tag{14}$$

$$\text{Since } \epsilon_s = z_1 \tan\theta = \frac{H}{2}\frac{\sigma_s}{\sigma_p}\tan\theta$$

$$\epsilon_z = \frac{H}{2}\tan\theta\left(\frac{\sigma_s}{\sigma_p} + 1\right) \tag{15}$$

Most experiments have shown that the longitudinal strain, ϵ_y, at the mid-point of the long side of the plate is very small compared to the transverse strain at that point. Thus, ϵ_z was taken as the principal or transverse strain at the point of expected failure. Again using a biaxial strain relation as given in Appendix C, the uniaxial strain is,

$$\epsilon_t = \frac{\epsilon_z}{1-\nu^2} \tag{16}$$

where ν is equal to Poisson's Ratio.

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7 Young in reference E5 recommended the use of σ_p in the equation for the shift of the neutral axis due to the addition of membrane stresses.

In order to find the stress at the expected point of failure, the uniaxial strain was used in the constituative relation for the plate material. The total stress will be designated σ_t and may be found for the strain ϵ_t from the stress-strain diagram of the material in simple tension.

The energy absorbed per unit volume at the mid-point of the long side will then be represented by:

$$U = \epsilon_t \sigma_t \left(\frac{\text{in-lbs}}{\text{in}^3}\right) \tag{17}$$

This value may be found at any point in the loading sequence if it is assumed that θ, the angle of rotation of the cross-section, is equal to ψ , the angle of the slope of the deflection profile at the edge of the plate as shown in Figure 3. This is a reasonable, as well as a convenient assumption, since the slope at the edge is determined during the deflection calculations. Thus, for each value of F and its corresponding W,

$$\tan \theta = \frac{dZ}{dx} \tag{18}$$

Then ϵ_t may be rewritten as

$$\epsilon_t = \frac{H}{2(1-\nu^2)} \left(\frac{dZ}{dx}\right)\left(\frac{\sigma_s}{\sigma_p} + 1\right) \tag{19}$$

Also, the stress-strain relations for the plate material that are required for the deflection relations may be used to find σ_t. Therefore, for any loading and corresponding deflection, the energy absorbed by the plate at the mid-point of the long side may be determined. A comparison of this energy to the restraint energy of the material will lead to a failure criterion.

Failure Criterion

When a plate is drawn in biaxial tension, the strain rate becomes unstable when the principal stress reaches the ultimate strength of the material. The plate begins to decrease in thickness rapidly like a tension specimen begins to neck at the ultimate stress.[P1] For this reason it was decided to use the ultimate stress of the plate material to establish the failure criterion. The ultimate stress will be defined as the stress at which the slope of an engineering stress-strain diagram is equal zero. The area under the stress-strain curve for all strains less than the strain at the ultimate stress will represent the stability of the plate material.

A restraint energy relation will be defined as

$$R = \epsilon_u \sigma_u \left(\frac{\text{in-lbs}}{\text{in}^3} \right) \tag{20}$$

where σ_u is the ultimate stress and ϵ_u is the ultimate strain. Both values are defined at the point of zero slope of the stress-strain curve.

The term restraint energy is used since R represents the amount of energy that might be absorbed by the material before instability begins. A more accurate measure of the ability of the material to absorb energy would be the area under the stress-strain curve. Since this area is usually difficult to obtain readily, the R relation is used. It provides a reasonable representation of the area under most stress-strain curves. Furthermore, it is more desirable to obtain a relation that will give a consistent measure of the restraint energy rather than an accurate one.

The two parameters that best correlate the minimum amount of failure data that is available are the deflection to span ratio, W/A, and the absorbed energy to restraint energy relation U/R. These non-dimensional parameters represent virtually all of the factors involved in the plate problem. Several other parameters were considered, but there did not appear to be any relation between them and the failure data. The W/A ratio helps to bring together all the geometric properties of the plate. The material properties are well represented by the U/R ratio.

Figure 5 is a plot of the two non-dimensional parameters for most of the rectangular plate failure data that is available. The curves are drawn through data points obtained by the calculation procedure described in the previous sections. The failure points for each plate are designated in two positions. The calculations for the deflections of the plate did not correspond directly with the experimental data as given in references E2 and E3. In general, the calculated deflections were slightly lower than the experimental deflections for a given total load. The possible reasons for this discrepancy will be discussed later. It was felt that both failure deflection and failure load should be indicated in Figure 5. Thus, the first set of failure points were found by using the experimental deflection at the mid-point of the plate to determine the W/A ratio at failure. The second set of points were obtained by using the failure load from the experimental data. For each rupture pressure, the total loading force F was calculated and from Figure 8, the corresponding U was found. The U/R values for the failure loads are shown on Figure 5 as the lower set of failure points on the curves.

The lower set of failure points was chosen to represent the failure data. Although the lower set provides the more conservative choice for a failure criterion, the major reason for using this set is that the failure data corresponds better to the deflection and absorbed energy calculations described in the previous sections.

A line can be drawn on Figure 5 connecting most of the failure points. With the exception of the plate designated 1B this line represents a minimum relation of W/A to U/R for plate failure. If it is moved slightly to the left, the failure point for plate 1B will also lie above the line. This new position of the limit line will be used as the failure criterion.

If δ = W/A and ϕ = U/R, then failure may be expected beyond the line defined by

$$\delta \approx .278 - 0.182\ \phi \tag{21}$$

Greenspon in reference T4 used essentially the same failure data and stated that failure should be expected for loads beyond the pressure that produces W/A ≈ 0.10. This limit as shown in Figure 5 is quite conservative for $\phi < 0.9$, but for plates with $\delta < 0.10$ and $\phi > 0.9$ the limit may be too large.

The energy relation was chosen for this analysis in order to provide a parameter that will accurately represent all types of loading conditions. Unfortunately, the available failure data is for loads uniformly distributed over the entire plate. In order to properly relate the failure for partial loads and uniform loads, equivalent absorbed energy relations are used. Thus the failure criterion should hold for all but concentrated loads with small bearing surfaces.

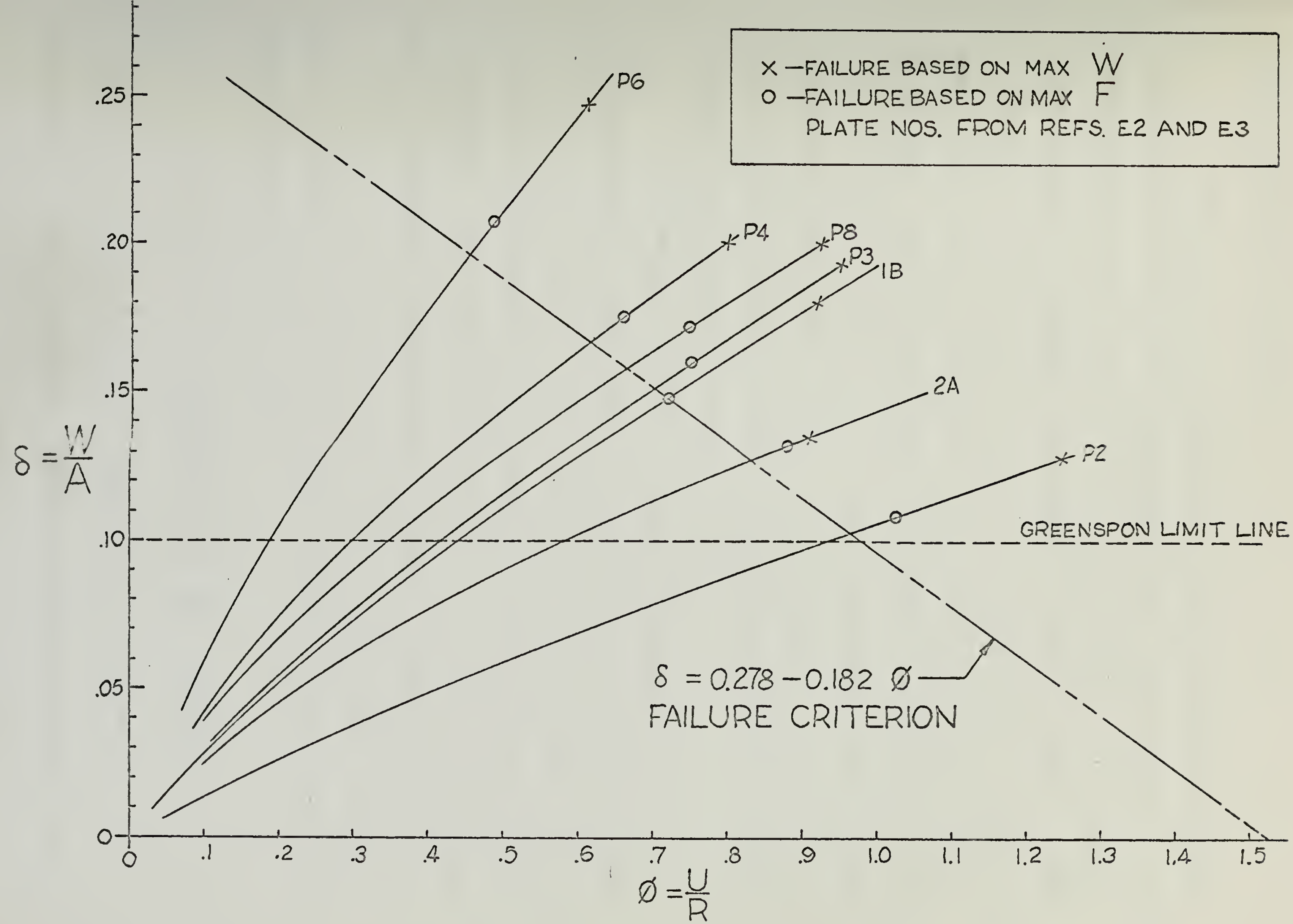


Figure 5. Plate Failure Parameters δ and ϕ for the Experimental Plates of References E2 and E3.

Summary of Method of Analysis

The method of calculation of the deflection and the absorbed energy for any plate has been incorporated into a short computer program. The description of the program itself is in Appendix E. The only difference between the procedure summarized below and the computer program is the method of determining a stress for a given strain. A generalized equation for the plastic portion of the stress-strain curve is used in the program. With the proper choice of variables the generalized stress-strain equation can represent the actual stress-strain curve of a given material with reasonable accuracy. There are several variations to the general equation that may be used. The relation described in Appendix E appeared to be one of the easiest to use.

The computer provided the speed and accuracy to carry out the calculations for many plates. The hand calculations are not difficult, but they are quite tedious. With the use of the tables and curves presented in the Appendixes, the time for calculation can be shortened considerably. The step-by-step calculations for a plate of known geometric and material properties and a given ice-crushing pressure are:

1. For the values of B/A, A/H, and σ_o/P_c enter the curves given in Figures A3, A4 or A5 to obtain the value of C/A at the collapse load. Then calculate $F_o = C^2 P_c$.

2. Calculate $Z = \frac{4 C^2 k P_c}{\pi^3 S}$, and $\frac{dZ}{dx} = \frac{P_c C k_d}{2S}$,

and $W = Z + \frac{1}{2}(A-C)\frac{dZ}{dx}$ where k and k_d may be

obtained from Figure B1 for B/A. Also the membrane force for the first

approximation will be assumed to equal the yield stress times the thickness.

3. The quantities W/A, B/A, and C/A are required to find the value of the average transverse strain at the mid-point of the plate. Tables of values of the average transverse strain and representative curves are given in Appendix D.

4. The new value of membrane stress will be found from the stress-strain curve for the plate material for the average transverse strain found in step number 3. Then $S_i = \sigma_s H$.

5. Steps 2, 3, and 4 are now repeated as described on page 28 until the new value of S_i is within an accepted difference from the previous value. (An absolute difference of 25 psi is used as a tolerance in the computer program).

6. For the final values of σ_s and dZ/dx from step 5, calculate $\epsilon_t = \frac{H}{2(1-\nu^2)}\left(\frac{dZ}{dx}\right)\left(\frac{\sigma_s}{\sigma_p} + 1\right)$.

7. Again enter the stress-strain curve for the plate material to find σ_t which corresponds to ϵ_t.

8. Calculate the energy absorbed at the point of expected failure. $U = \epsilon_t \, \sigma_t$

9. Use the value for ϵ_u as defined on page 33, and calculate $R = \epsilon_u \, \sigma_u$.

10. Using δ = W/A and ϕ = U/R, solve the failure criterion equation $FC = \delta + .182\,\phi - .278$. If $FC \geq 0$, then failure can be expected for any additional loading.

11. If $FC \leq 0$, increase the total load F by the load increment ΔF which may be any predetermined amount. Obtain a new value for $C = \sqrt{F/P_c}$

12. Repeat steps 2 through 11 to find the deflection and absorbed energy values for the new loading condition.

It is not necessary to evaluate the possibility of failure at each load increment. After one or two times through the procedure the approximate point of failure may be apparent and the solution method can be shortened.

RESULTS OF APPLICATIONS OF THE METHOD OF ANALYSIS

Plastic Collapse Load and Deflection

The method for determining the total load for plastic collapse depends on a given ice crushing pressure. The area over which this pressure must act to cause collapse is determined as outlined on page 22. If this method is reversed so that for a given C/A and A/H a certain σ_o/P_c is found, then the required pressure for collapse will be known. When this pressure is compared to elastic experimental data for plates of the same size and material properties, the collapse pressure should be considerably greater than the elastic loading pressure. It is very difficult to compare deflections, although the deflection should be increased by an amount similar to the loading.

The plastic collapse load and deflection were calculated for plates of the same size and material properties as those used by Sturm and Moore (E4). The size of the bearing surface was fixed at 2" x 2" and 12" x 12". Then the plastic collapse load was found from Figures A3-A5. In this case P_c, which has been called the ice crushing pressure, is the collapse load. Finally, using these values for C and P_c the collapse deflection, W_o, was determined by steps 2-5 of the previous section. The results of the calculations for three of the plates are shown in Table 1 along with experimental data from reference E4.

A second comparison of the plastic collapse loads and deflections was made with the results from the large deflection theory of plates. This provided a means of judging the accuracy of the membrane solution method for calculating the plastic collapse deflections for

uniformly loaded plates.

Reference E8 compares many of the solutions to the large deflections of plates to experimental results. Most of the relations are given in the form of curves for the non-dimensional deflection, stress, and pressure ratios. The deflections of three plates were calculated from the corresponding pressure ratio. The plates were of the same size and material as those given in Table 1. The collapse pressure obtained from Figures A3-A5 for C/A = 1.0 was used in the pressure ratio,

$$P.R. = \frac{p_o A^4}{16EH^4} \tag{22}$$

For example, Figure A3 shows that $\sigma_o/P_c = 215$ for an aluminum ($E = 10 \times 10^6$ psi) plate 1"x48"x48" with $\sigma_o = 38800$ psi and C/A = 1.0. Thus $P_c = p_o = 180.5$ psi. Using the value in the pressure ratio, a deflection ratio W/H = 0.9 was obtained from Figure 24 of reference E8. The same procedure was followed for the other two plates. The results from the large deflection theory and the membrane solution for collapse deflections and loads are shown in Table 2.

The Membrane Solution for Plate Deflections

It was relatively easy to determine the accuracy of the membrane solutions for the plate deflection. The plate dimensions and material properties of the test plates used by Day [E2,E3] and Loeser [E1] were put into the computer program to determine the load/deflection relationships. The results of the calculations as well as the input data are shown in Tables E1 to E11 of Appendix E. It should be noted

2" x 2" LOAD BEARING SURFACE

PLATE SIZE	σ_o (psi)	F max (lbs)		W max (in)		σ max (psi)	
		Exptl.	Calc.	Exptl.	Calc.	Exptl.	Calc.
1"x48"x96"	38800.0	10000.0	31050.0	0.225	3.68	19800.0	46000.0
1"x48"x48"	38800.0	10000.0	25900.0	0.181	2.36	18400.0	43250.0
½"x48"x48"	38500.0	2000.0	10300.0	0.294	1.98	15900.0	41250.0
12" x 12" LOAD BEARING SURFACE							
1"x48"x96"	38800.0	10000.0	55800.0	0.20	1.17	9200.0	38800.0
1"x48"x48"	38800.0	10000.0	69800.0	0.163	1.04	9700.0	38800.0
½"x48"x48"	38500.0	2000.0	16910.0	0.252	0.511	7600.0	38500.0

Table 1. Comparison of Calculated Plastic Collapse Deflections with Experimental Results of Reference E4.

PLATE SIZE	σ_o (psi)	Plastic[1] Collapse Pressure (psi)	Defl. Ratio W/H		Collapse Defl. (in)	
			L.D.T.[2]	Memb.[3]	L.D.T.	Memb.
1"x48"x96"	38800.0	106.5	0.9	.721	0.9	.721
1"x48"x48"	38800.0	180.5	0.9	.787	0.9	.787
½"x48"x48"	38500.0	45.2	1.8	.794	0.9	.397

1 See example page 41 of text.

2 L.D.T. = Large Deflection Theory as given in Figure 24 of reference E8.

3 Memb. = Membrane Solution as described in steps 1-5 on pages 37 and 38 of text.

Table 2. Comparison of Calculated Plastic Collapse Deflections with Calculated Deflections from Large Deflection Theory for a Uniformly Distributed Load.

that the input variable for ice crushing pressure for these calculations is only the loading pressure that is used for the first calculation. The pressure used for any additional loading is equal to the applied force divided by the plate area. It was necessary to follow this procedure because all of the test data is for uniformly distributed loads. Actually the solution method is the same as if an ice-load is used and C/A = 1.0. In this way the computer program may be used for any type of lateral load except highly concentrated loads.

Also, the first calculation for deflection given in Tables E1 to E11 does not represent the collapse deflection. The starting point used for these calculations was chosen for convenience in comparing the results to the respective experimental data. For example, the rupture pressure given in Day's experiments was divided by five and the result was used as the input for ice crushing pressure in the computer program. Then the increment of applied force ΔF was set equal to the ice crushing pressure times the plate area.

The results of the calculations are shown in Figures 6 and 7. The experimental results of Loeser and Day are also plotted in each figure.

Energy Absorbed Relation

It was of interest to plot curves of total load F to the energy absorbed U for three sets of data. The F versus U relations for the calculations on Day's test plates as given in Tables E1 to E7 are shown in Figure 8. Similar curves for calculations on Loeser's test plates (Tables E8 to E11) are plotted in Figure 9. The results

of calculations made on arbitrary plates under idealized ice-loads are given in Tables E12 to E19, and the F versus U curves are shown in Figure 10.

The calculations for Loeser's and Day's test plates have been previously described. The results shown in Tables E12 to E19 are for four plates with the same B/A ratio, but of varying thickness. Also, two sets of material properties were used for each plate. The calculations follow the format as described in the method of analysis and Appendix E.

A particular case is encountered in the calculations for the arbitrary plates that was bypassed for the calculations of Loeser's and Day's test plates. The assumption that the ice crushing pressure changes when $C/A = 1.0$ has been discussed previously. The analysis of the arbitrary plates required this change. This can be seen by tracing down the C/A column in Tables E12 to E19. For all but the thicker plates C/A starts at the value for $(C/A)_0$ and increases until $C = A$. For square plates the ice crushing pressure will change to the total load divided by the plate area. But for plates where $B/A > 1.0$, this change can cause a decrease in the crushing pressure rather than the desired increase. As a consequence the deflection will be smaller than the previous deflection. In order to help eliminate this unnatural discontinuity, it was assumed that the loading surface continued to grow after $C/A = 1.0$. That is, the loading pressure was kept constant until not just the width of the plate but the entire plate was covered. Then for any further increases in the load, the pressure increased as the force divided by the area of the plate. This scheme would be the

natural procedure to follow if the calculations were being done by hand but it required a few extra statements in the computer program.

Application of Failure Criterion

The relation between $\delta = W/A$ and $\phi = U/R$ has been plotted in Figure 5 for the results shown in Tables E1 to E7. The failure criterion was established from these curves and the failure points given by Day [E2,E3]. The establishment of the δ versus ϕ curves is discussed on pages 33-35.

For the W and U values given in Tables E8 to E11, the corresponding δ's and ϕ's were calculated. The relation between the two failure parameters is shown in Figure 11. These curves represent the experimental work of Loeser [E1].

Similarly, Figure 12 is a plot of δ and ϕ for the calculations made on the arbitrary plates given in Tables E12 to E19. The arbitrary plates by their sizes and properties represent typical hull plating on ice-strengthened ships. Thus, the curves of Figure 12 give a good indication of the possibilities of failure of these plates under the idealized ice-loads.

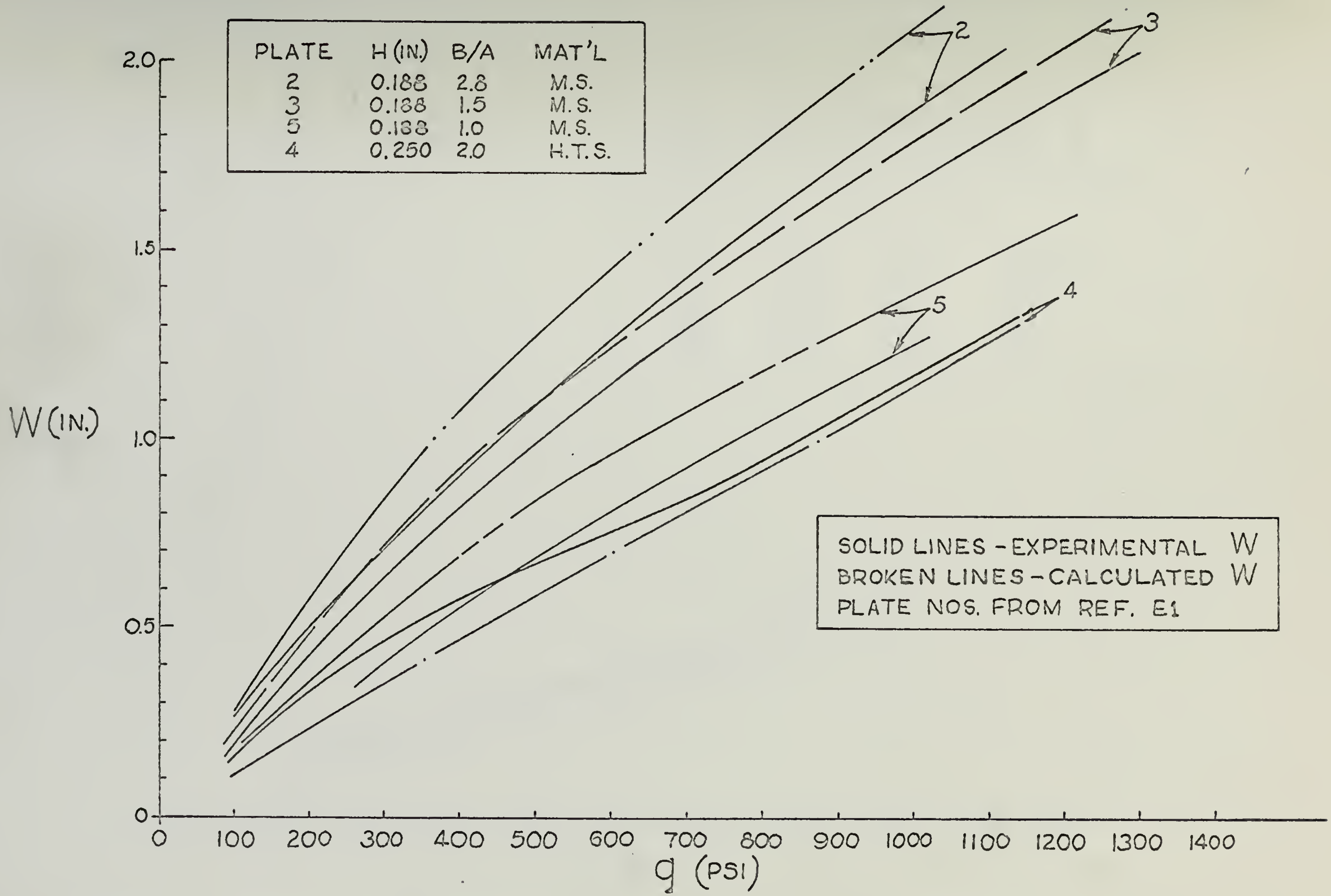


Figure 6. Comparison of Plate Deflections to the Experimental Results of Loeser (E1)

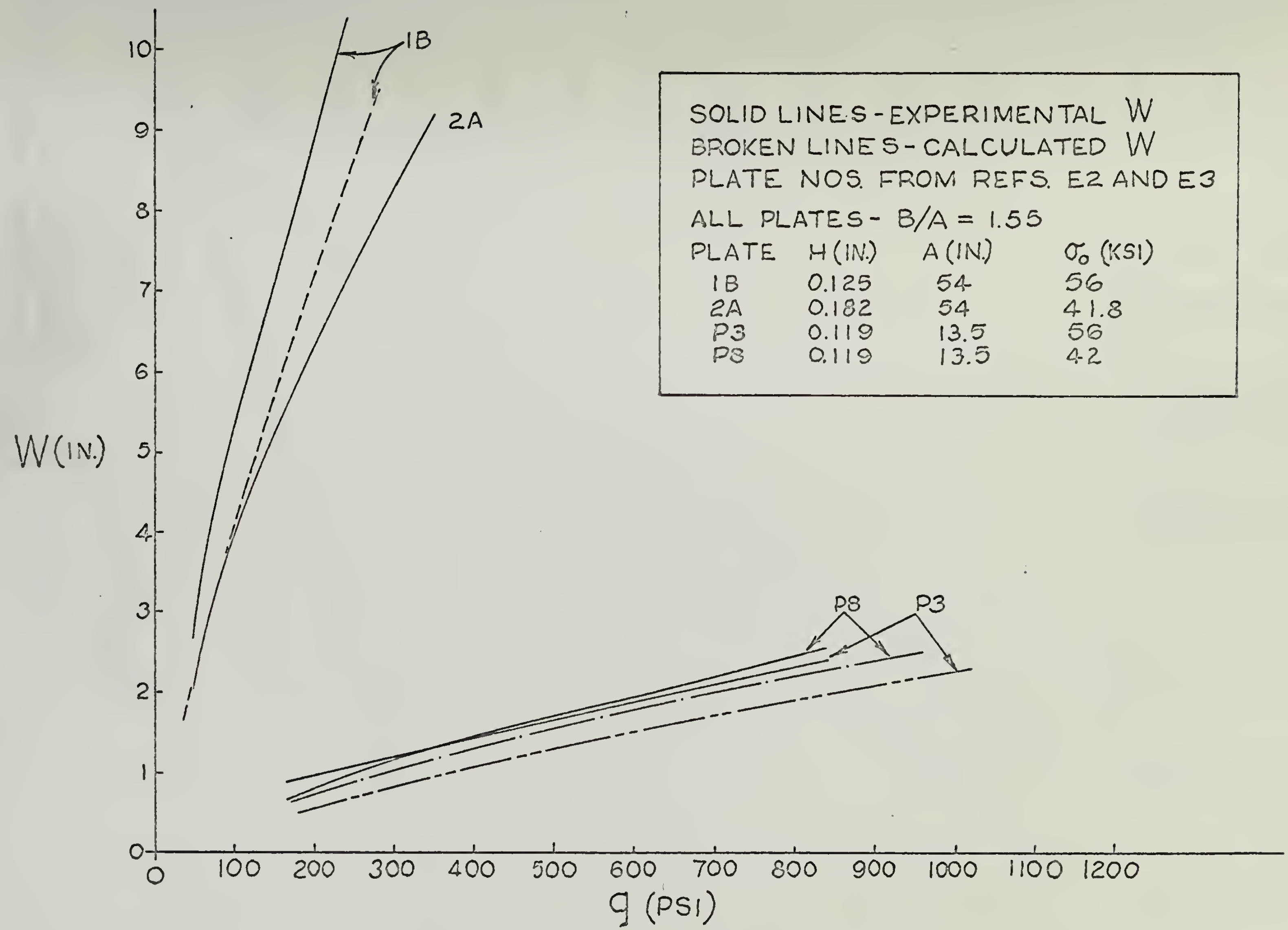


Figure 7. Comparison of Calculated Plate Deflections to the Experimental Results of Day (E2,E3)

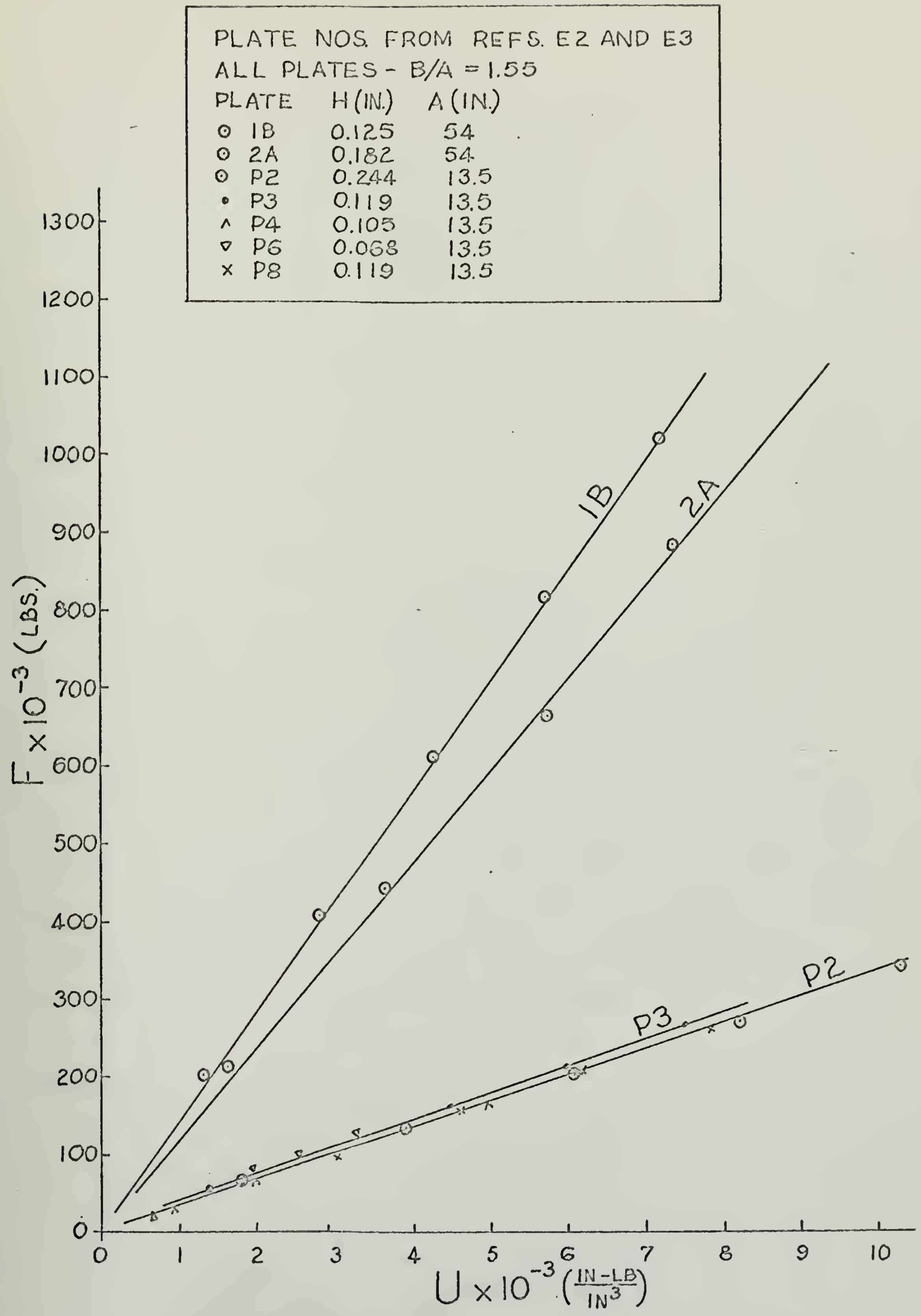


Figure 8. Load versus Absorbed Energy for Day's (E2, E3) Experimental Test Plates

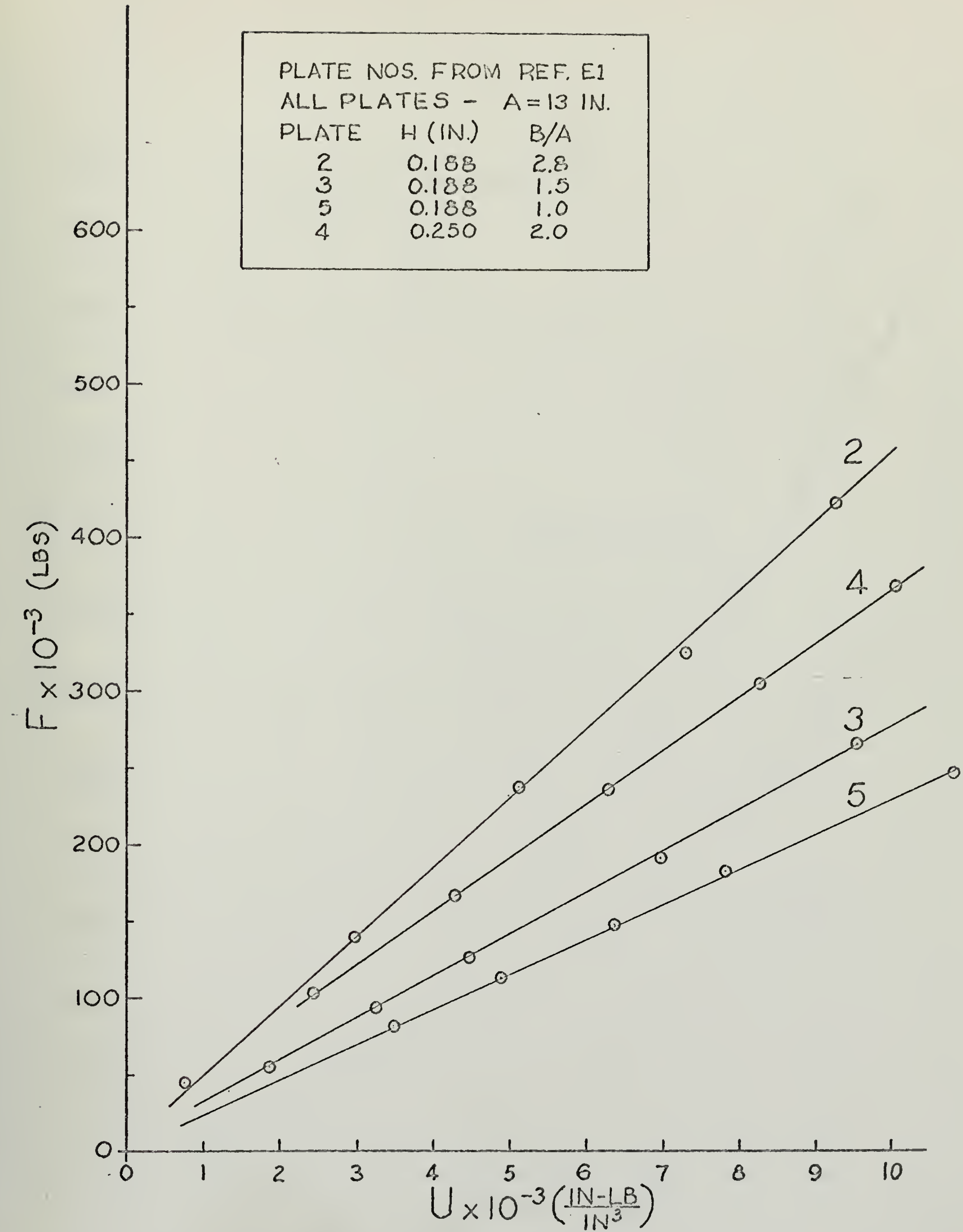


Figure 9. Load versus Absorbed Energy for Loeser's (E1) Experimental Test Plate

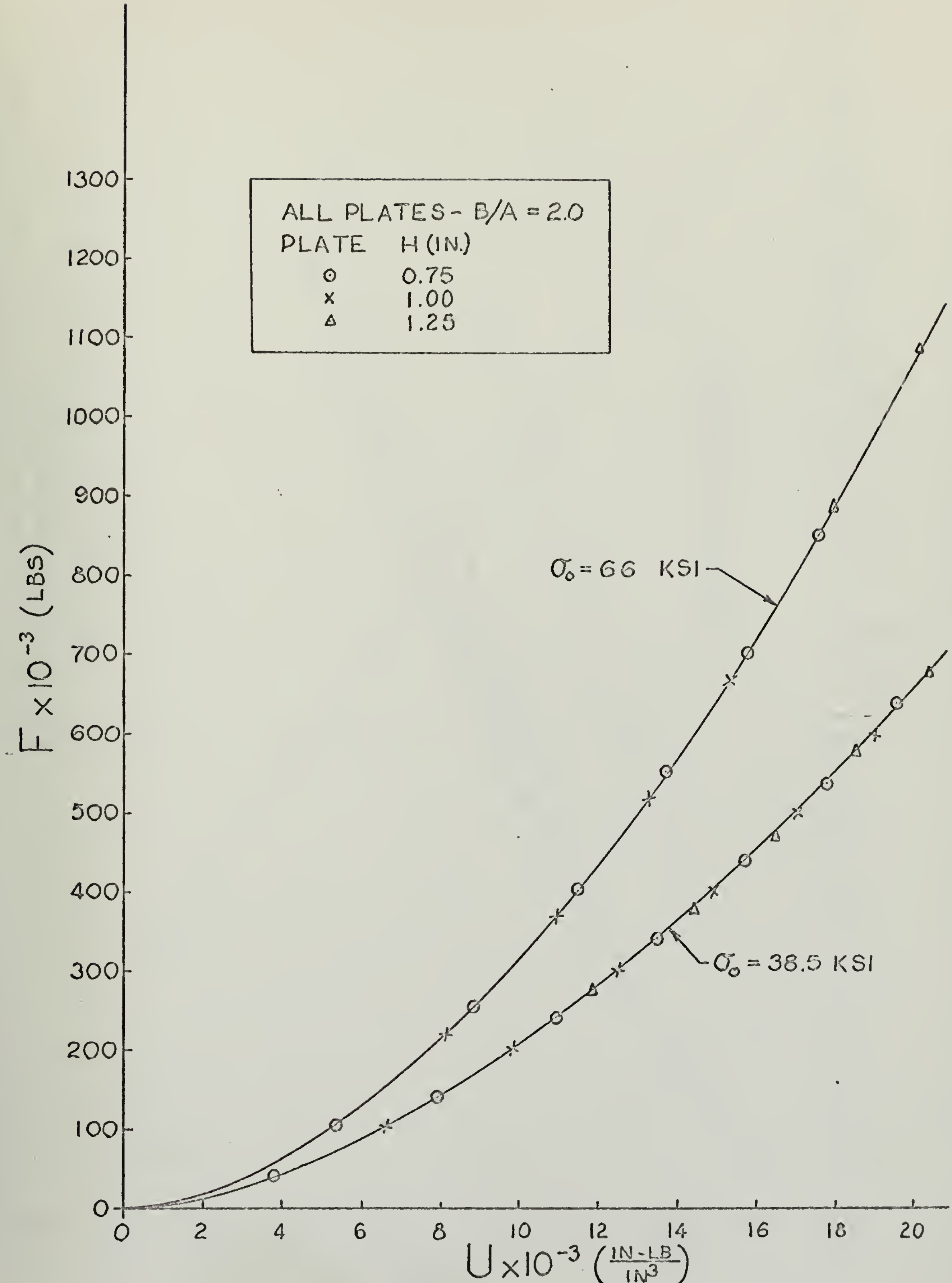


Figure 10. Load versus Absorbed Energy for Arbitrary Plates of Tables E12 to E19

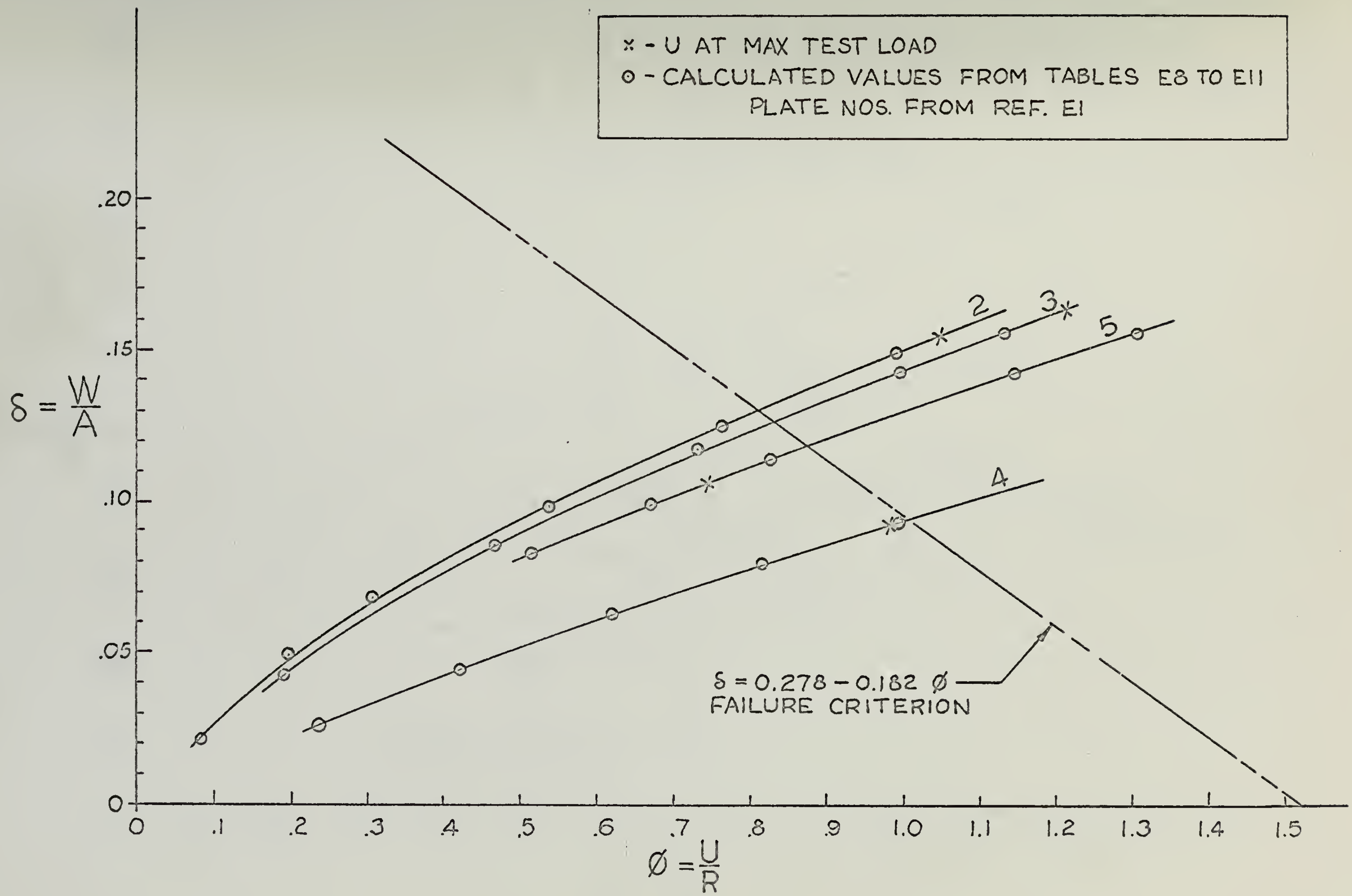


Figure 11. Plate Failure Parameters δ and ϕ for the Experimental Plates of Reference E1

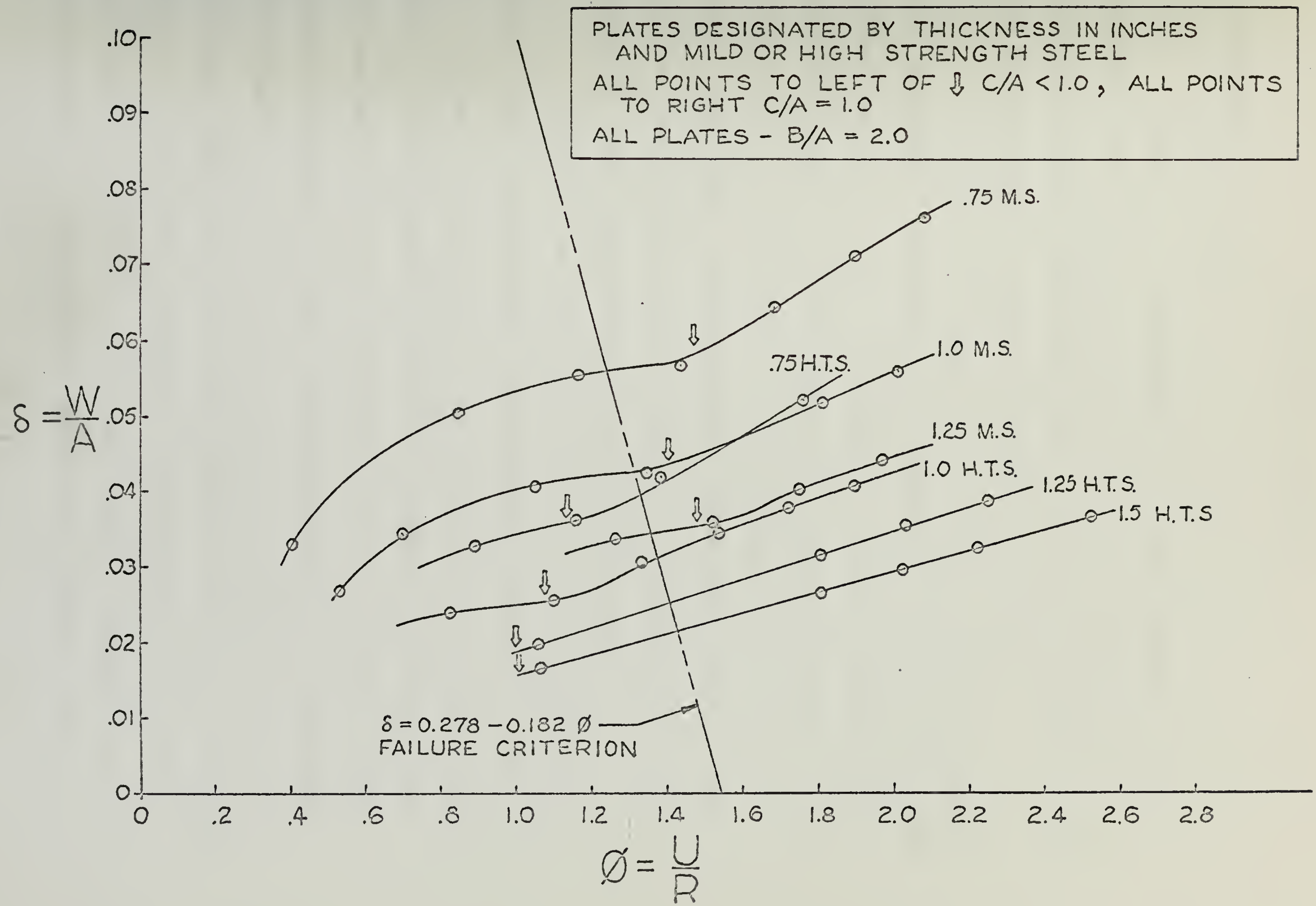


Figure 12. Plate Failure Parameters δ and ϕ for the Arbitrary Plates of Tables E12 to E19

DISCUSSION OF RESULTS

Load/Deflection Relations

Figures 5 to 11 and Tables 1 and 2 are presented so that a comparison of the various portions of the method of analysis may be made with associated experimental data. The degree of accuracy of each step of the analysis is shown in the tables and figures.

Although the data presented in reference E4 was published in 1937, it proved to be the most useful experimental data on rectangular plates under partial loads. The comparison between Sturm and Moore's experiments and the calculated values for collapse deflection and load given in Table 1 is very crude. The experimental data is for the elastic range only, and the calculated values assume that the plastic collapse load has been reached. However, the experimental data does provide at least a lower bound to the calculated values.

There are two general points that should be recognized from Table 1. The relation between the deflections and the B/A ratio that is indicated by the experimental data holds for the calculated values. That is, the experimental deflections increase as B/A increases and the same holds true for the calculated values. A similar comparison between deflections and plate thicknesses shows that the experimental deflections increase as the plate thicknesses decrease. However, the calculated deflections do not follow the same trend as the experimental results. It seems that the thinner plate should have the greatest deflection as it does in the experimental data. Although only relative comparisons have been made, the failure of the membrane solutions for

the collapse deflections to compare favorably with the experimental results should be considered a limitation of the method of analysis.

A comparison of the plastic collapse deflection obtained from the membrane solution with the deflections calculated from large deflection theory may be made from Table 2. In general, the membrane solution deflections are lower than those obtained with the same pressure from large deflection theory. For this case of uniformly distributed loads, both sets of calculations show that the deflection increases as the B/A decreases. Also, the deflection for the large deflection theory remains constant for the three plates considered. Again the relation between deflections and plate thicknesses does not compare favorably. The membrane solution maintains approximately the same W/H ratio at collapse for all thicknesses. If the results from the large deflection theory are used to define the plastic collapse condition, then the limitation in the method of analysis as discussed for Table 1 is again encountered.

The accuracy of the membrane solution may be easily seen in Figures 6 and 7. The calculated deflections are either completely above the experimental deflections or completely below. In all cases, if the plate has strength properties (see Tables E1 to 11) greater than mild steel (σ_o = 38000 psi) the deflections lie below the experimental data. For the higher strength plates, the deflections are all below the experimental results. The one exception is Day's test plate 2A in Figure 7. It has a yield strength slightly above the mild steel plates and the curves for the calculated and experimental deflections coincide.

Since the shapes of the calculated and experimental deflection curves agree very well, the only major discrepancy is the difference in

the deflections as noted above. The reason for the differences is the solution method for obtaining the membrane force for the membrane formula. This same reason may be cited as the cause for giving lower values of deflection for high strength materials and higher deflections for lower strength materials. The generalized stress-strain curve used in the computer program usually gives values of stress that are lower than the actual value for the same strain. This will in turn produce larger membrane deflections since the membrane force is $S = \sigma_s H$. Furthermore, for two plates of approximately the same thickness, the membrane stress will be greater for a plate made of high strength metal than one with a lower yield strength. Consequently, the membrane deflections for the high strength material will be less than those of materials with a lower yield stress.

For the general discussion of the application of the method of analysis, Figures 13 and 14 illustrate what the load/deflection relationship is for hull plating under the idealized ice-loads. The curves and the respective results of the computer calculations given in Tables E12 to E19 show that for small load bearing surfaces the collapse deflections appear to be excessive. The collapse deflections for the large bearing surfaces are usually smaller than what might be expected. For the uniformly loaded plates, the collapse deflection is not necessary for the load deflection analysis. Thus, for plates that require a collapse load greater than that given by step 1 of the method of analysis, the first value for deflection given in Tables E15, E18, and E19 does not represent the collapse deflection.

For the partial loading conditions there is no direct relation

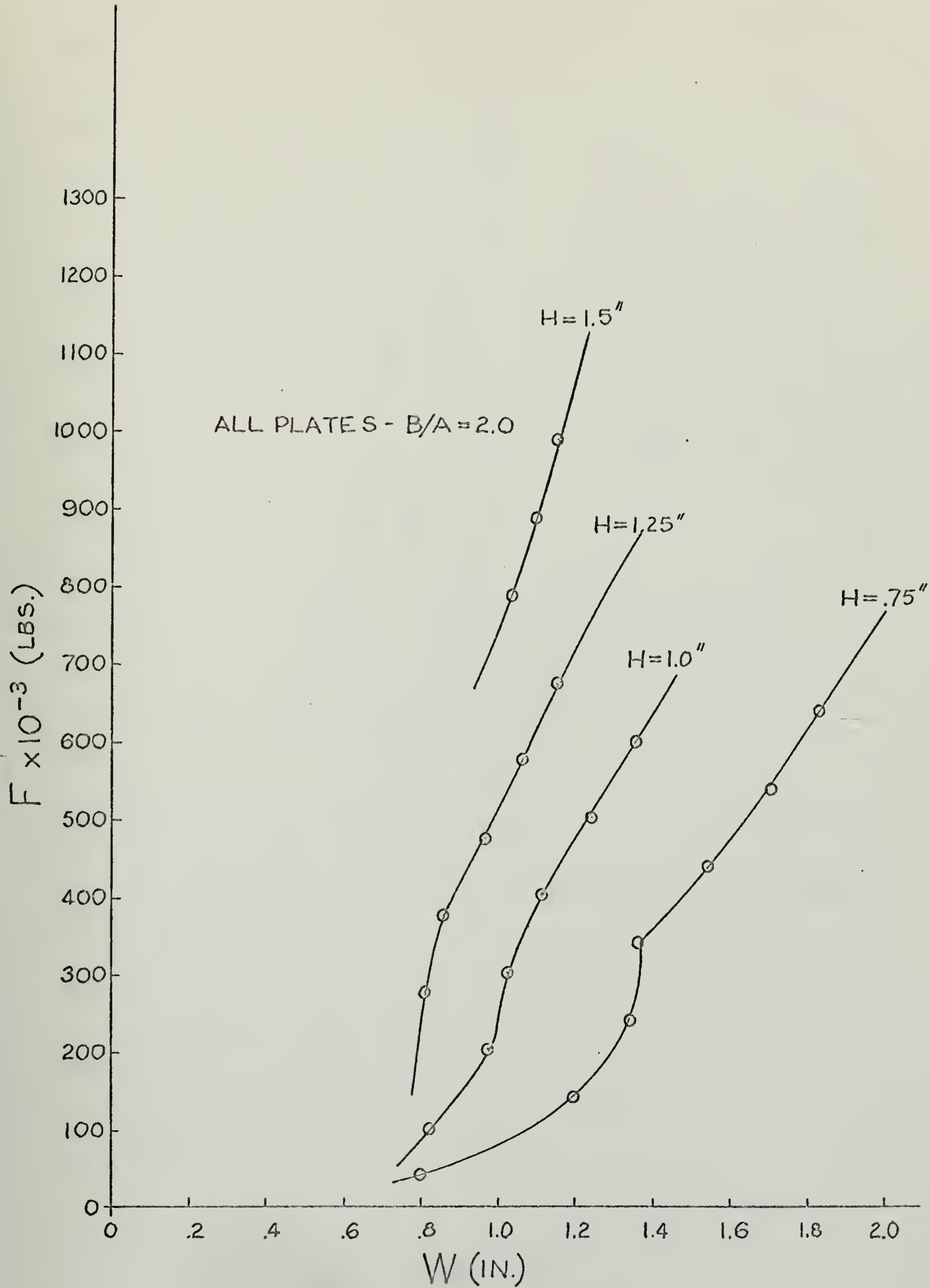


Figure 13. Load/Deflection Relation for the Arbitrary Plates of Tables E12 to E14 (Mild Steel Material Properties)

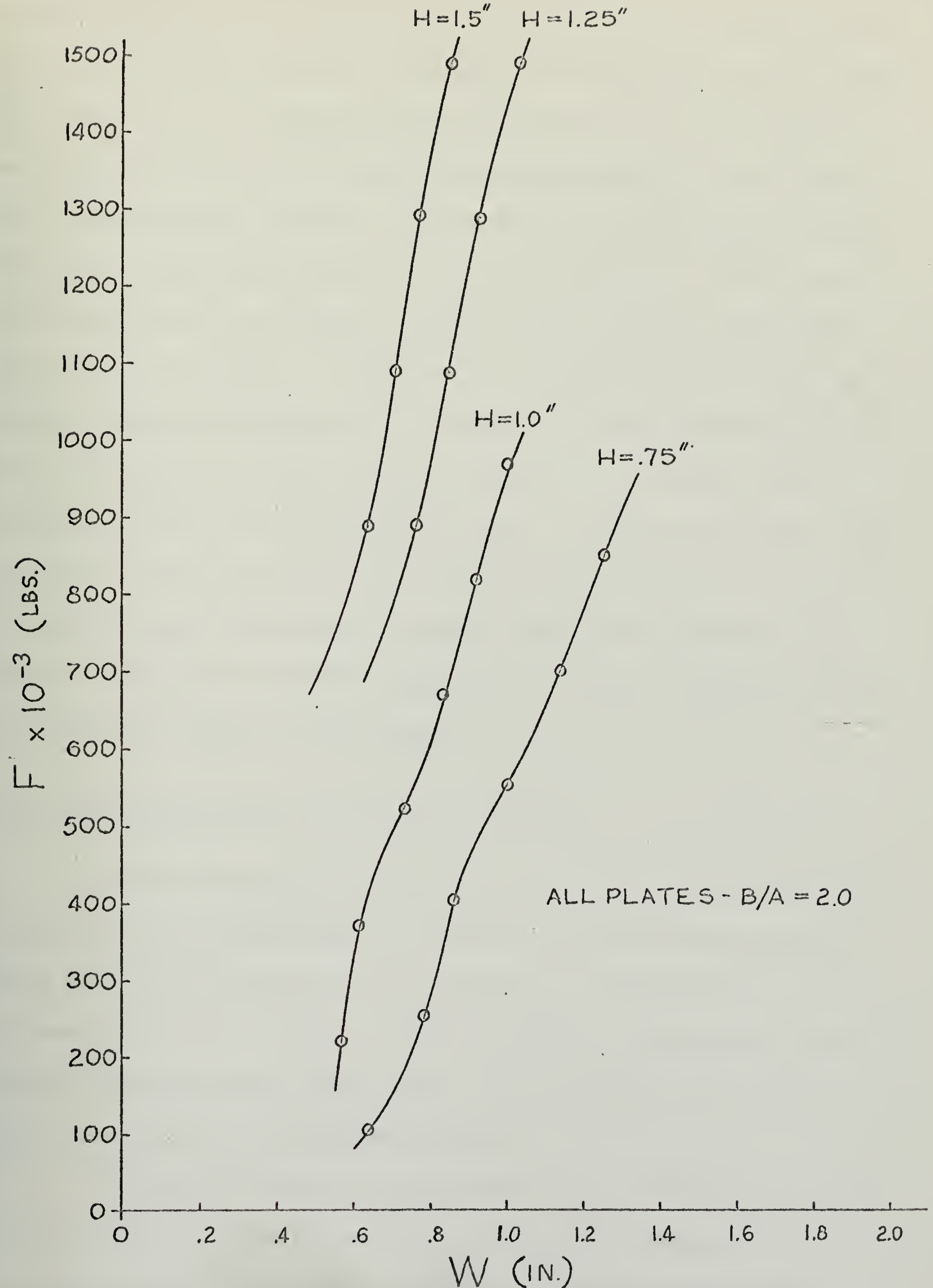


Figure 14. Load/Deflection Relation for the Arbitrary Plates of Tables E15 to E19 (High Strength Material Properties)

between the deflection at one value of the applied force and the next. Thus, once the uniform load is reached, the deflection should be within the same accuracy as the plates shown in Figures 6 and 7. This fact alone should help to substantiate the loading process. In all cases, the deflection/load relation, in general, follows a continuous curve. There are exceptions to this, but they are caused by the change over from the constant ice crushing pressure to the uniformly distributed load. If, for example, the deflection for the first partial load is greater than the deflections for subsequent loadings, the deflection will be decreasing as the total load increases. As can be seen in Figures 13 and 14, the deflection is never less than the previous value. Therefore, if the accuracy of the calculated deflections for the uniformly distributed loads can be accepted, then the calculated deflections for the partial loadings should be of reasonable values for use in the failure analysis of the plates.

Absorbed Energy Relation and Failure Criterion

There is one basic purpose for calculating the energy absorbed by the plate during the loading operation. The calculated energy provides a means of determining the possibility of plate failure on any increase in the load. There are several methods of determining the amount of energy absorbed by a plate. The method presented provides a measure at the point of expected failure.

It has been found in experimental tests of dynamically loaded clamped plates (see references E19 and E20) that a linear relation exists between the measure of applied force and the measure of the energy

absorbed by the plate. The relation between F and U in Figures 8 and 9 is clearly linear for all plates. The results shown in Figure 10 for the arbitrary plates under the idealized ice-loads are not linear. The nonlinearity is due to the changing size of the bearing surface whereas the curves of Figures 8 and 9 are for a uniformly distributed load throughout the loading process.

It is interesting to note that the only difference between the two curves in Figure 10 is the material properties of the plates. The F versus U curve for each of the four plate thicknesses considered coincides with the other plates of the same material properties. This same general relation appears to hold for the plates shown in Figures 8 and 9, although B/A ratios and plate sizes differ in these figures. Thus, based on the results of Figures 8 and 9, and the consistency of the results shown in Figure 10, the absorbed energy relation, U, should provide a good measure of the possible failure of a plate.

The calculated values for δ and ϕ from the data given in Tables E8 to E11 are plotted in Figure 11. The values of U representing the maximum load that was applied to each of the experimental plates by Loeser [E1] are also indicated. With the exception of plate number five, all of the maximum load points lie on or above the failure criterion line. Plate number five failed due to a weld defect, but the other plates did not fail in Loeser's experiments. Thus, there appears to be at least a rough correlation of the failure criterion for two different experimental results.

Figure 12 shows the final application of the method of analysis. If the line of the criterion is accepted for the smaller values of δ

then the thicker plates can be expected to fail shortly after their collapse deflection is reached. This is actually what would be expected for the very stiff plates. However, it seems that the line of the failure criterion should include larger values of ϕ for the smaller values of δ. This is just a speculation, and the presently defined failure criterion appears to provide a reasonable estimate of the failure load of a plate.

CONCLUSIONS

The method of analysis for large deflections of a clamped rectangular plate under idealized ice-loads provides a reasonable means of predicting the behavior of a plate. Furthermore, the solution method provides a simultaneous check on the possibility of plate failure. The calculations are tedious, but not difficult, and the use of the computer program will greatly aid any extensive failure analysis.

A failure criterion was sought and obtained. Furthermore, the criterion is not limited to plates under ice-loads. It should be equally applicable to the analysis of a plate under any lateral load except in cases where shear effects are large.

RECOMMENDATIONS

The major recommendation is quite obvious, that is, to conduct tests to provide more data for the failure criterion. Other areas that should be investigated are other possible solution methods for the load/deflection relations and the energy absorbed relation. These areas should be included in any failure tests since they are necessary for the establishment of the failure criterion.

Appendix A

PLASTIC COLLAPSE LOAD

The elastic moment at the center of a partially loaded simply supported rectangular plate is given by Timoshenko in reference T1. The moment about the longitudinal axis at the mid-point is,

$$M_x = \beta P \tag{A1}$$

where $P = qC^2$ and β is given in Tables 20-22 of reference T1. The values of β depend on C/A and B/A, and those values given in Tables 20-22 decrease in accuracy as C/A decreases. Values for β for the smaller values of C/A can be found from the equation for M_x given on p. 160 of reference T1. That equation is:

$$M_x = \frac{P}{8\pi}\left[\left(2\log\frac{4A}{\pi C\sqrt{2}} + \lambda - 1.571\right)(1+\nu) + \mu(1-\nu)\right] \tag{A2}$$

The parameters λ and μ depend on B/A and the values for them are given in Table 27 on p. 161 of reference T1. The values of β calculated from equation (A2) for C/A less than 0.20 are given in Table A1.

In order to use equation (A1) for a plate with a clamped edge condition, the coefficient β must be adjusted, since the values given in Tables 20-22 of reference T1 and Table A1 are for a simply supported plate. The correction term β_1 may be found on p. 206 of reference T1 for various B/A ratios. Thus, the maximum moment at the center of a clamped rectangular plate under a partial load is,

$$(M_x)\ \max = (\beta + \beta_1)\ P = \beta' P \tag{A3}$$

The factor β' is plotted against C/A for three values of B/A in Figure A1. Two steps were necessary in finding the proper value of β'. First, β was plotted against C/A for the values of β given

in Tables 20-22 of reference T1 and for those values of β given in Table A1. Then for each B/A, a single curve was formed which combined the two solutions of β . An example of the graphical solution for β is shown in Figure A2. The second step required adding the constant β_1 to the new values of β obtained graphically to complete the solution of β' .

Once the elastic solution for the moment at the mid-point of a clamped rectangular plate was established, the plastic collapse load was found by simple plastic analysis. The plastic moment at a cross-section through the thickness of a rigid, perfectly plastic plate is

$$M_o = \frac{\sigma_o H^2}{4} \qquad (A4)$$

Setting (M_x) max $= M_o$ the plastic collapse load becomes,

$$P_o = \frac{\sigma_o H^2}{4 \beta'} \qquad (A5)$$

Since $P_o = q_o C^2 = P_c C^2$ for an ice crushing pressure equal to P_c

$$C^2 = \frac{\sigma_o H^2}{4 \beta' P_c}$$

or

$$C = \frac{H}{2}\sqrt{\frac{\sigma_o}{\beta' P_c}} \qquad (A6)$$

By dividing through by the width of the plate, A, equation (A6) may be non-dimensionalized.

$$\frac{C}{A} = \frac{H}{2A}\sqrt{\frac{\sigma_o}{\beta' P_c}}$$

or

$$\frac{\sigma_o}{P_c} = 4 \beta' \left[\left(\frac{A}{H}\right)\left(\frac{C}{A}\right)\right]^2 \qquad (A7)$$

The desired relation between the load bearing surface and the ratio of material and ice properties for given plate geometry may be obtained from

equation (A7).

The parameter σ_o/P_c for values of B/A, C/A, and A/H has been tabulated in Tables A2, A3, and A4. From these values the curves of A/H versus σ_o/P_c for various values of C/A have been drawn in Figures A3-A5. It was found that this was the best way to present the data in order to make use of them in the plate analysis. For a rectangular plate of given dimensions and yield stress being loaded by ice with a given crushing pressure, $(C/A)_o$, the size of the load bearing surface at the plastic collapse load may be found for A/H and σ_o/P_c from the curves of Figures A3-A5.

$\beta \quad (\nu = 0.3)$

C/A \ B/A	1.0	1.2	1.4	1.6	1.8	2.0	3.0	∞
0.02	0.3605	0.3778	0.3889	0.3957	0.3998	0.4022	0.4052	.4054
0.04	.3045	.3218	.3329	.3397	.3438	.3462	.3492	.3494
0.06	.2725	.2898	.3009	.3077	.3118	.3142	.3172	.3174
0.08	.2495	.2668	.2779	.2847	.2888	.2912	.2942	.2944
0.10	.2315	.2488	.2599	.2667	.2708	.2732	.2762	.2764
0.15	.1995	.2168	.2279	.2347	.2388	.2412	.2442	.2444
0.20	.1765	.1938	.2049	.2117	.2158	.2182	.2212	.2214

Table A1. Load Coefficients β for M_x of Equation (A2).

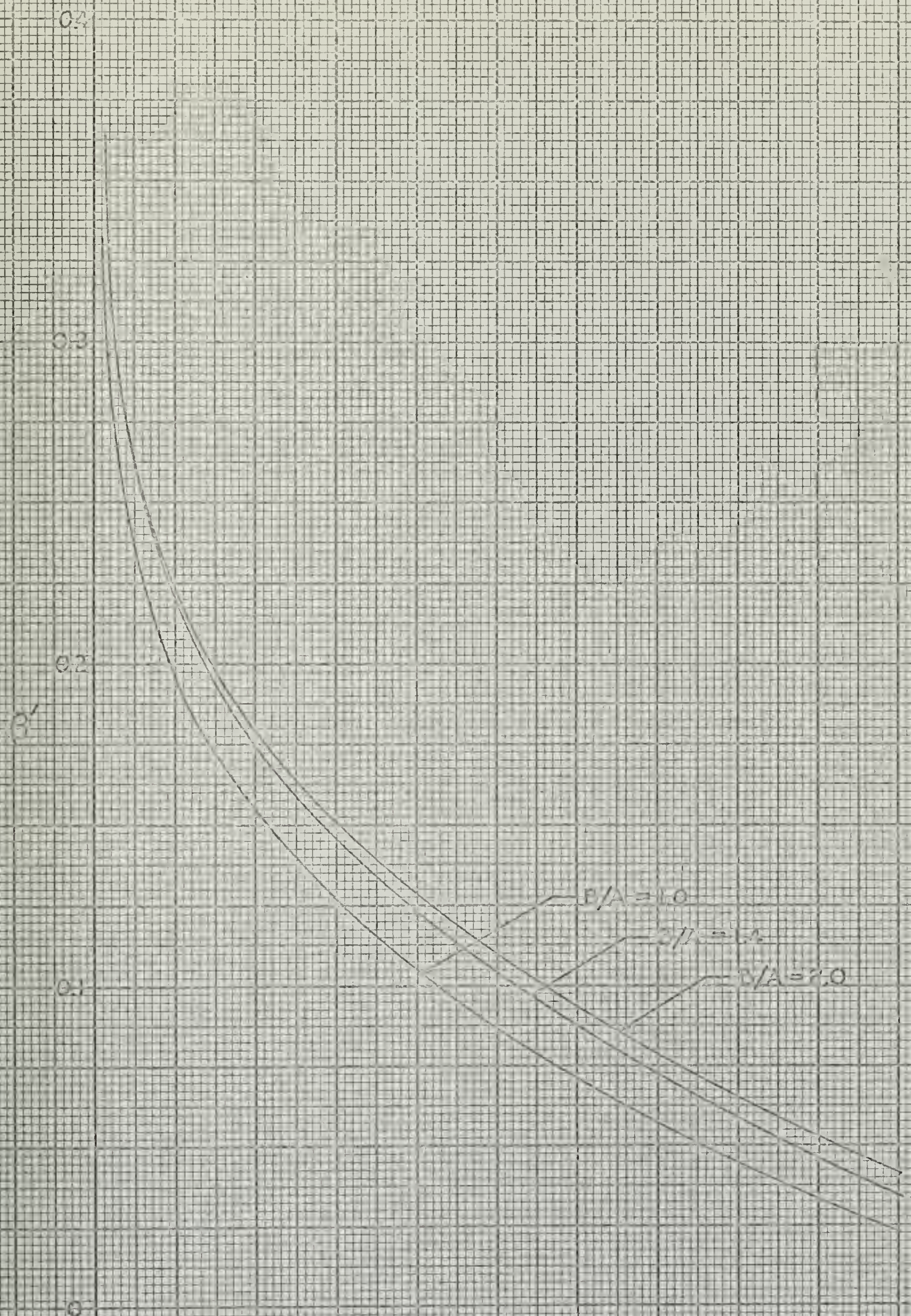


Figure A1. Load Coefficient β' for the Bending Moment at the Mid-Point of a Partially-Loaded Clamped Rectangular Plate (See equation A3)

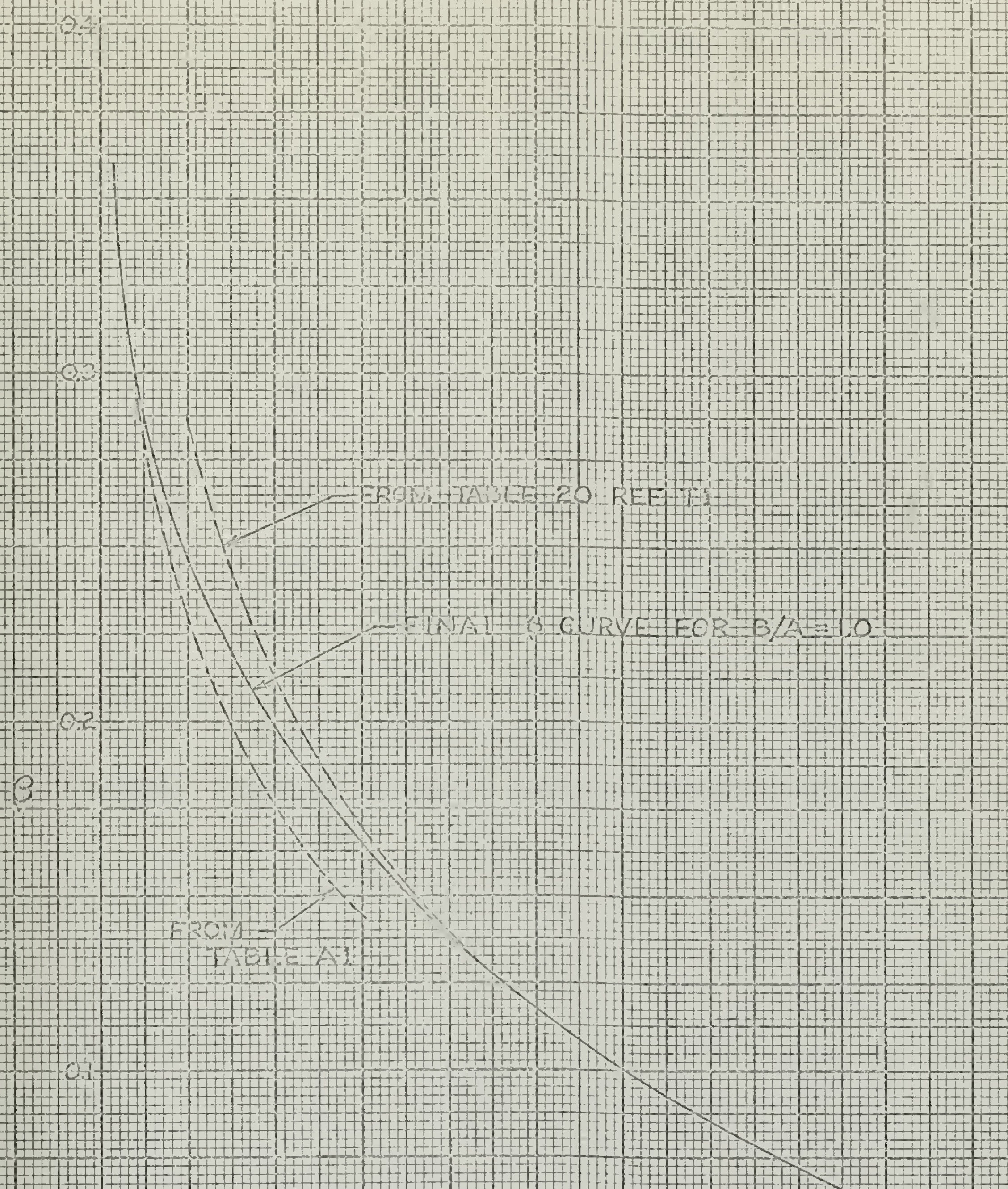


Figure A2. Example of the Graphical Solution for the Load Coefficient β in Equation A3

RATIO OF YIELD STRESS TO ICE CRUSHING PRESSURE

B/A= 1.000

A/H	10.0	20.0	30.0	40.0	50.0	100.0
C/A						
0.020	0.0	0.2	0.4	0.8	1.2	4.9
0.040	0.2	0.6	1.4	2.6	4.0	16.1
0.060	0.3	1.3	3.0	5.3	8.2	32.9
0.080	0.5	2.1	4.8	8.6	13.4	53.6
0.100	0.8	3.1	7.1	12.6	19.6	78.6
0.200	2.5	9.9	22.3	39.7	62.0	248.0
0.400	6.8	27.1	61.1	108.5	169.6	678.4
0.600	10.4	41.8	94.0	167.0	261.0	1044.0
0.800	11.5	46.1	103.7	184.3	288.0	1152.0
1.000	9.2	37.0	83.2	147.8	231.0	924.0

Table A2

RATIO OF YIELD STRESS TO ICE CRUSHING PRESSURE

B/A= 1.400

A/H	10.0	20.0	30.0	40.0	50.0	100.0
C/A						
0.020	0.1	0.2	0.5	0.8	1.3	5.2
0.040	0.2	0.7	1.6	2.8	4.3	17.4
0.060	0.4	1.4	3.2	5.7	8.9	35.7
0.080	0.6	2.4	5.4	9.6	15.0	60.0
0.100	0.9	3.5	8.0	14.2	22.1	88.5
0.200	2.8	11.2	25.2	44.8	70.0	280.0
0.400	7.8	31.4	70.6	125.4	196.0	784.0
0.600	12.4	49.5	111.5	198.1	309.6	1238.4
0.800	14.8	59.1	132.9	236.3	369.3	1477.1
1.000	14.0	55.8	125.6	223.4	349.0	1396.0

Table A3

RATIO OF YIELD STRESS TO ICE CRUSHING PRESSURE

B/A= 2.000

A/H	10.0	20.0	30.0	40.0	50.0	100.0
C/A						
0.020	0.1	0.2	0.5	0.8	1.3	5.3
0.040	0.2	0.7	1.6	2.8	4.4	17.6
0.060	0.4	1.5	3.3	5.9	9.1	36.6
0.080	0.6	2.4	5.5	9.7	15.2	60.7
0.100	0.9	3.6	8.1	14.3	22.4	89.6
0.200	2.9	11.5	25.9	46.1	72.0	288.0
0.400	8.2	32.8	73.7	131.1	204.8	819.2
0.600	13.4	53.6	120.5	214.3	334.8	1339.2
0.800	16.6	66.6	149.8	266.2	416.0	1664.0
1.000	16.4	65.6	147.6	262.4	410.0	1640.0

Table A4

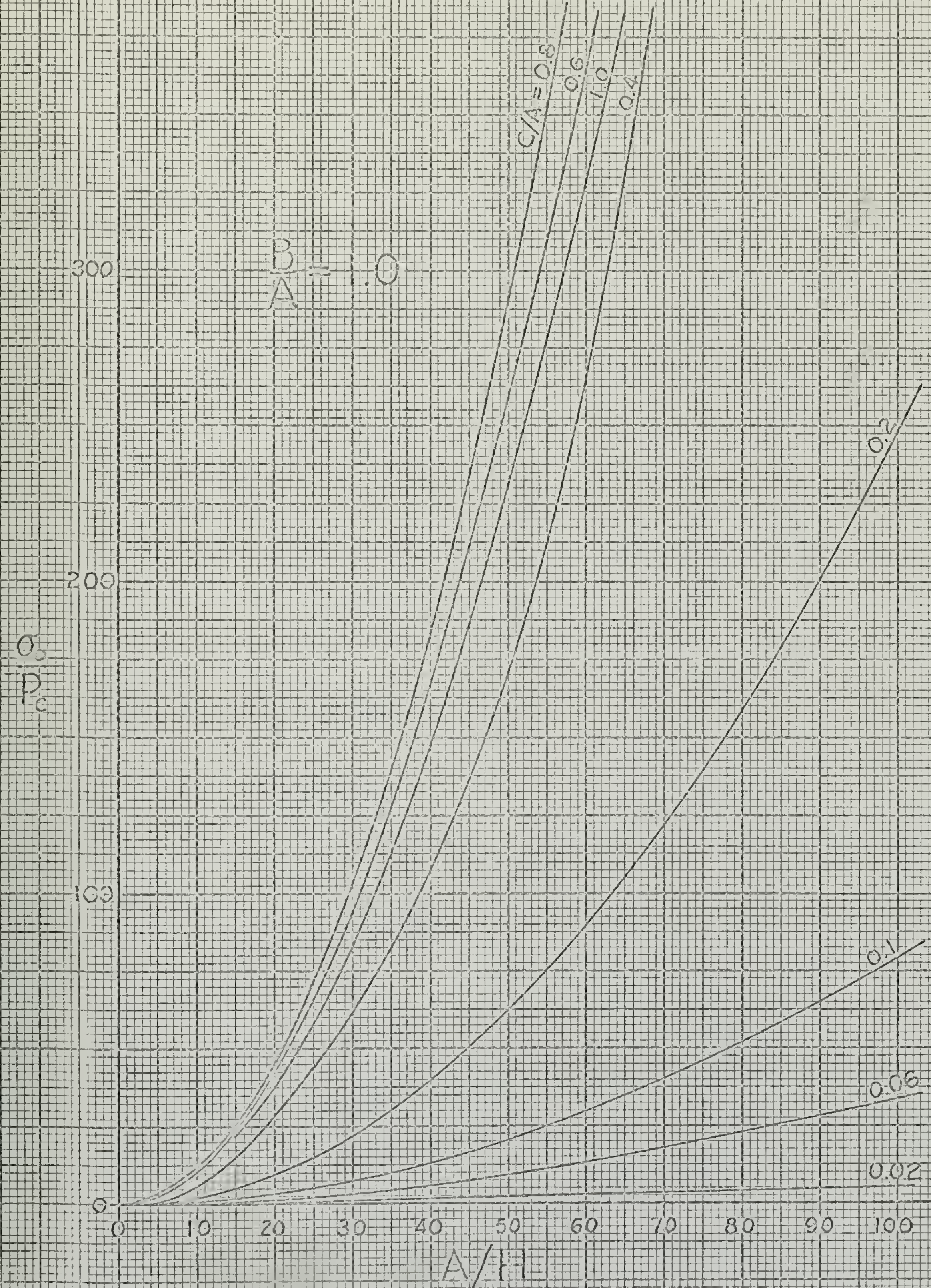


Figure A3. Size of the Load Bearing Surface at the Plastic Collapse Load for Values of B/A, A/H, and σ_o/P_c (See Equation A7)

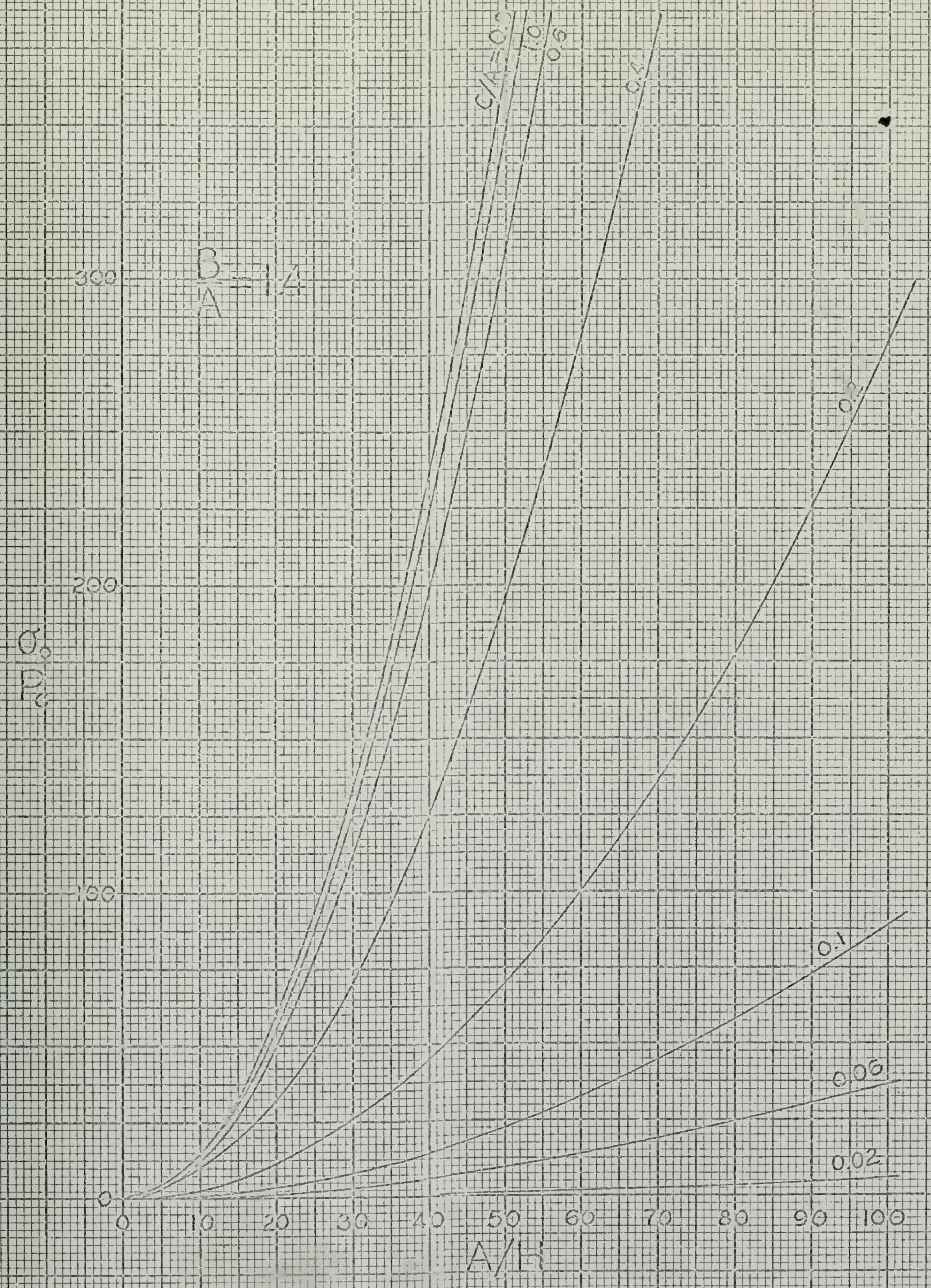


Figure A4. Size of the Load Bearing Surface at the Plastic Collapse Load for Values of B/A, C/H, and σ_o/P_c (See Equation A7)

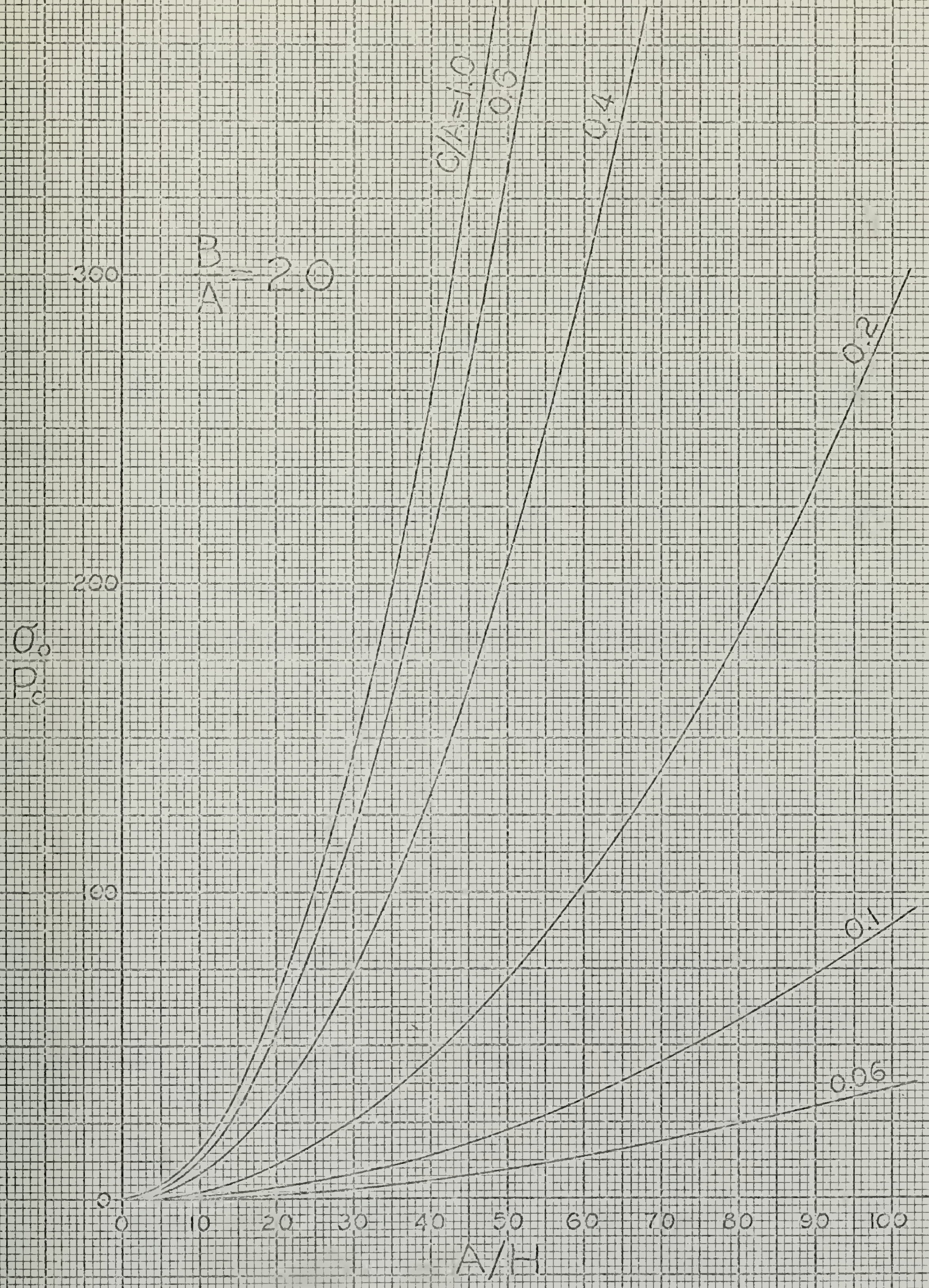


Figure A5. Size of the Load-Bearing Surface at the Plastic Collapse Load for Values of B/A, A/H, and σ_o/P_c (See Equation A7)

Appendix B

DERIVATION OF MEMBRANE FORMULA FOR PLATE DEFLECTION

The solution for the deflection of a uniformly loaded membrane is given in several references. (See for example, references T1, T4, and E1). The method presented here is the one given by Loeser.(E1) Since the method has been developed elsewhere, only the major points of the solution are given.

The differential equation of a uniformly loaded rectangular membrane is:

$$\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} = - \frac{q}{S} \tag{B1}$$

where q is the lateral loading pressure and S is the membrane force. For the plate dimensions given in Figure 2, the boundary conditions are:

$$Z = 0 \text{ at } X = 0,\ X = A,\ y = \pm \frac{B}{2}$$

The assumed solution for equation (B1) is the series:

$$Z = \sum_{n=1,3,5}^{\infty} b_n \sin \frac{n\pi x}{A} Y_n \tag{B2}$$

The b_n's are constant coefficients, and Y_n's are dependent upon y only. After differentiating twice with respect to x and y and substituting for $\frac{\partial^2 Z}{\partial x^2}$ and $\frac{\partial^2 Z}{\partial y^2}$, equation (B1) becomes:

$$\sum_{n=1,3,5}^{\infty} b_n \sin \frac{n\pi x}{A} \left(- \frac{n^2 \pi^2}{A^2} Y_n + Y_n'' \right) = - \frac{q}{S} \tag{B3}$$

Expansion of the right hand side of equation (B3) into a Fourier series and using constant value for q between $0 \leq x \leq A$ results in the following expression:

$$\frac{q}{S} = \frac{4q}{\pi S} \sum_{n=1,3,5}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n} \sin \frac{n\pi x}{A} \qquad \text{(B4)}$$

Setting equations (B3) and (B4) equal to each other gives

$$Y_n'' - \frac{n^2\pi^2}{A^2} Y_n = -\frac{q}{S}\frac{4}{n\pi b_n} (-1)^{\frac{n-1}{2}} \qquad \text{(B5)}$$

Solution of this ordinary differential equation for the conditions that Y_n is symmetric about $y = 0$ and that $Y_n = 0$ when $y = \pm B/2$ yields.

$$Y_n = \frac{4qA^2 (-1)^{\frac{n-1}{2}}}{S\, n^3 \pi^3 b_n \cosh \frac{n\pi B}{2A}} \left(\cosh \frac{n\pi B}{2A} - \cosh \frac{n\pi y}{A} \right) \qquad \text{(B6)}$$

Then the membrane deflection may be obtained by substituting equation (B6) into equation (B2).

$$Z = \sum_{n=1,3,5}^{\infty} \frac{4qA^2 (-1)^{\frac{n-1}{2}}}{S\, n^3 \pi^3} \left(1 - \frac{\cosh \frac{n\pi y}{A}}{\cosh \frac{n\pi B}{2A}} \right) \sin \frac{n\pi x}{A} \qquad \text{(B7)}$$

If the constant terms are removed and the series is defined as k, the membrane deflection equation is:

$$Z = \frac{4qA^2}{S\pi^3} k \qquad \text{(B8)}$$

and

$$k = \sum_{n=1,3,5}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^3} \sin \frac{n\pi x}{A} \left(1 - \frac{\cosh \frac{n\pi y}{A}}{\cosh \frac{n\pi B}{2A}} \right) \qquad \text{(B9)}$$

The values for k have been plotted in Figure B1 for three values of x/A. The use of these curves eases the burden of computation for the deflection. For example, the value of the central deflection, W, of a uniformly loaded plate will require the value of k corresponding to the B/A ratio of the plate and x/A = 0.50. This same value of k will hold for all values of q and S.

The slope of the membrane surface is obtained by differentiating equation (B8) with respect to x. Thus, the slope at the midpoint of the long side of the plate is:

$$\frac{\partial Z}{\partial x} = \frac{4qA}{S\pi^2} \sum_{n=1,3,5}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \left(1 - \frac{\cosh \frac{n\pi y}{A}}{\cosh \frac{n\pi B}{2A}}\right) \cos \frac{n\pi x}{A} \qquad \text{(B10)}$$

Then for y = 0 and x = 0 or x = A,

$$\frac{\partial Z}{\partial x} = \frac{4qA}{S\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \left(1 - \frac{1}{\cosh \frac{n\pi B}{2A}}\right) \qquad \text{(B11)}$$

Noting that $\sum_{n=1,3,5}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$, equation (B11) becomes

$$\frac{\partial Z}{\partial x} = \frac{qA}{2S} - \frac{4qA}{S\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2 \cosh \frac{n\pi B}{2A}}$$

or

$$\frac{\partial Z}{\partial x} = \frac{qA}{2S} k_d \quad \text{at } x = y = 0 \qquad \text{(B12)}$$

where

$$k_d = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2 \cosh \frac{n\pi B}{2A}}$$

The coefficient k_d is also given in Figure B1 for values of B/A.

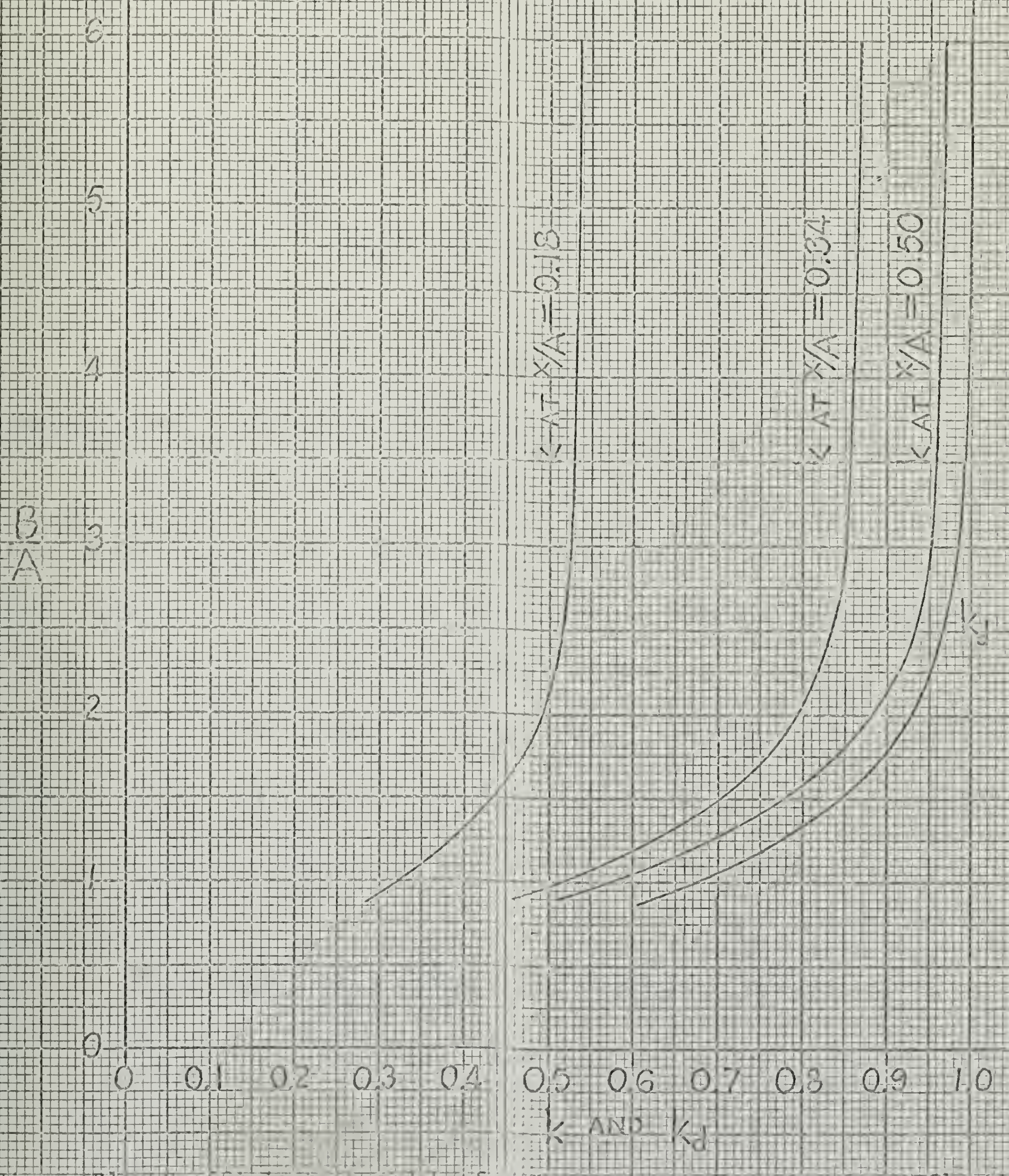
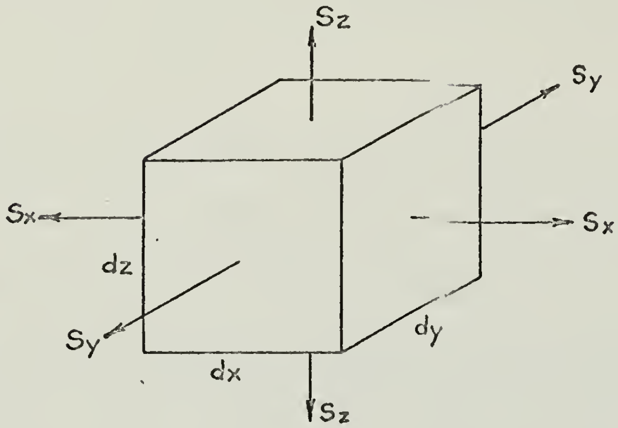


Figure B1. Coefficients k and k_d for the Membrane Equation for Deflection and the Derivative of the Equation (See Equations B8 and B12).

Appendix C

BIAXIAL STRAIN RELATION

Assume that a given element of the plate has only normal forces acting on each face.



For the case of uniaxial loading let $S_y = S_z = 0$. Then the strain components along each axis will be

$$\epsilon_{x1} = \epsilon_1, \quad \epsilon_{y1} = -\nu\,\epsilon_{x1}, \quad \epsilon_{z1} = -\nu\,\epsilon_{x1} \tag{C1}$$

where ϵ_1 represents the principal strain in the x direction. The strain components in the y and z directions are obtained by considering $S_x = S_z = 0$ and $S_x = S_y = 0$ respectively. Thus,

$$\epsilon_{y2} = \epsilon_2, \quad \epsilon_{x2} = -\nu\,\epsilon_{y2}, \quad \epsilon_{z2} = -\nu\,\epsilon_{y2} \tag{C2}$$

and

$$\epsilon_{z3} = \epsilon_3, \quad \epsilon_{y3} = -\nu\,\epsilon_{z3}, \quad \epsilon_{x3} = -\nu\,\epsilon_{z3} \tag{C3}$$

If all the forces are acting, the total strain in the x direction will be,

$$\epsilon_x = \epsilon_{x1} + \epsilon_{x2} + \epsilon_{x3} = \epsilon_1 - \nu\,\epsilon_{y2} - \nu\,\epsilon_{z3}$$

$$\epsilon_x = \epsilon_1 - \nu\,(\epsilon_{y2} + \epsilon_{z3}) \tag{C4}$$

For plates it is generally assumed that ϵ_z is small compared to the other strains. The total strain in the x direction for $\epsilon_z = 0$ will be

$$\epsilon_x = \epsilon_1 - \nu \epsilon_{y2} \tag{C5}$$

Similarly, the total strain in the y direction for $\epsilon_z = 0$ will be,

$$\epsilon_y = \epsilon_2 - \nu \epsilon_{x1} \tag{C6}$$

For the case where the in-plane strain is uniaxial in the x direction $\epsilon_y = 0$ and, hence, $\epsilon_2 = \nu \epsilon_{x1} = \nu \epsilon_1$. From equation (C2) $\epsilon_{y2} = \epsilon_2 = \nu \epsilon_1$. Substituting into equation (C5) $\epsilon_x = (1 - \nu^2) \epsilon_1$. (C7)

This relation will be useful in obtaining the stress at a given point where the strain in the x direction is known, and the strain in the y direction is zero. The stress can be found from a stress-strain diagram for simple tension by using

$$\epsilon_1 = \frac{\epsilon_x}{1 - \nu^2} \tag{C8}$$

A more general relation is required when there is strain in both the x and y directions. If $\epsilon_y \neq 0$, the strain in the x direction may be found by combining equations (C5) and (C6).

$$\epsilon_x = \epsilon_1 - \nu \epsilon_{y2} = \epsilon_1 - \nu \epsilon_2$$

$$= \epsilon_1 - \nu (\epsilon_y - \nu \epsilon_1)$$

$$\epsilon_x = (1 - \nu^2) \epsilon_1 - \nu \epsilon_y \tag{C9}$$

When used in the form

$$\epsilon_1 = \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2} \tag{C10}$$

the stress in the x direction may be found from a stress-strain diagram for simple tension.

Appendix D

PLATE DEFLECTION PROFILE AND AVERAGE TRANSVERSE STRAIN

In order to use the membrane formula for the calculation of plate deflections, a relation between the deflection and the membrane force must be established. This is accomplished by assuming a plate deflection profile. Then, for a given deflection, the average strain within the plate can be calculated. From the stress-strain curve for the average strain, a value for the in-plane or membrane stress can be found which is converted to the membrane force by multiplying by the plate thickness.

For this analysis, it was desired to use an assumed deflection profile that would represent the various loading conditions. Thus, the solution for a partially loaded simply supported rectangular plate as derived by Timoshenko [(T1)] was selected. The general expression from page 138 of reference T1 for the deflection at any point along the x axis (see Figure 2) is:

$$w = \frac{4qA^4}{D\pi^5} \sum_{m=1,3,5}^{\infty} \frac{(-1)^{\frac{m-1}{2}}}{m^5} \sin\frac{m\pi C}{2A} \left\{ 1 - \frac{1}{\cosh\alpha_m} \left[\cosh(\alpha_m - 2\gamma_m) + \gamma_m \sinh(\alpha_m - 2\gamma_m) + \alpha_m \frac{\sinh 2\gamma_m}{2\cosh\alpha_m} \right] \right\} \sin\frac{m\pi x}{A} \quad \text{(D1)}$$

where $\alpha_m = \frac{m\pi B}{2A}$ and $\gamma_m = \frac{m\pi C}{4A}$.

Since only the shape defined by this expression is desired, the factors in front of the summation sign were dropped. Then, in order to obtain

a non-dimensional profile, each value of deflection is divided by the deflection at $x = A/2$, the mid-point of the plate. The result is a series of shape coefficients which depend on B/A, C/A, and x/A.

The calculations for w/W for various values of B/A, C/A, and x/A were made much easier with the aid of the digital computer. The short program used to make the calculations is included at the end of this appendix. Table D6 is a list of the input variables required for the program. This same program is incorporated into the overall ice/plate interaction program discussed in Appendix E. The output from the plate deflection profile and transverse strain computer program is given in Tables D1 to D5. A typical plot of w/W versus x/A is shown in Figure D1. The variance in deflection profile for a concentrated load (C/A = 0.02) and a uniform load (C/A = 1.0) is illustrated in the figure. Other assumed deflection profiles are also plotted in order to provide a comparison with other membrane solutions.

The only reason for assuming a deflected shape of the plate is to determine the average strain within the plane of the plate. This is accomplished by measuring the length of the deflection profile, subtracting the original length of plate, and dividing the difference by the original length. If L equals the length of the transverse deflection profile, and A is the width of the plate, then the average transverse strain is:

$$\epsilon_x = \frac{L-A}{A} \quad \text{(in/in)} \tag{D2}$$

In order to determine L from the w/W, x/A relations, the deflection profile was divided into a series of straight lines. This

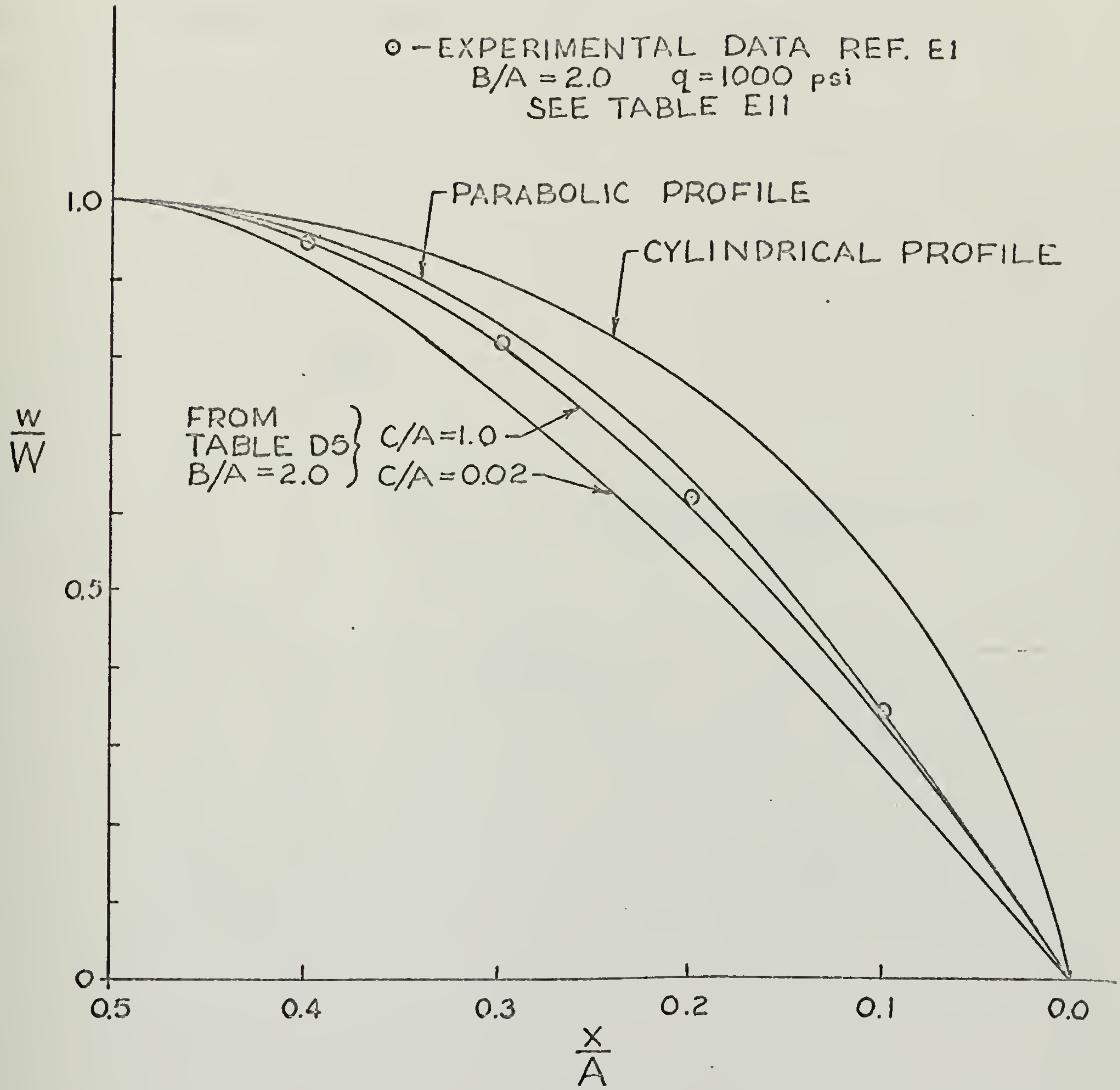


Figure D1. Deflection Profile along the Transverse Axis of a Partially Loaded Simply Supported Plate

linear approximation of the curve allowed rapid computation of L by the computer. In addition, the parameter W/A must be introduced so that the strain calculation is related to the actual plate deflection. Figure D2 represents a certain segment of the profile curve. By multiplying each segment of $\Delta(w/W)$ by W/A, the arc length ΔL will represent an actual segment of the deflection profile.

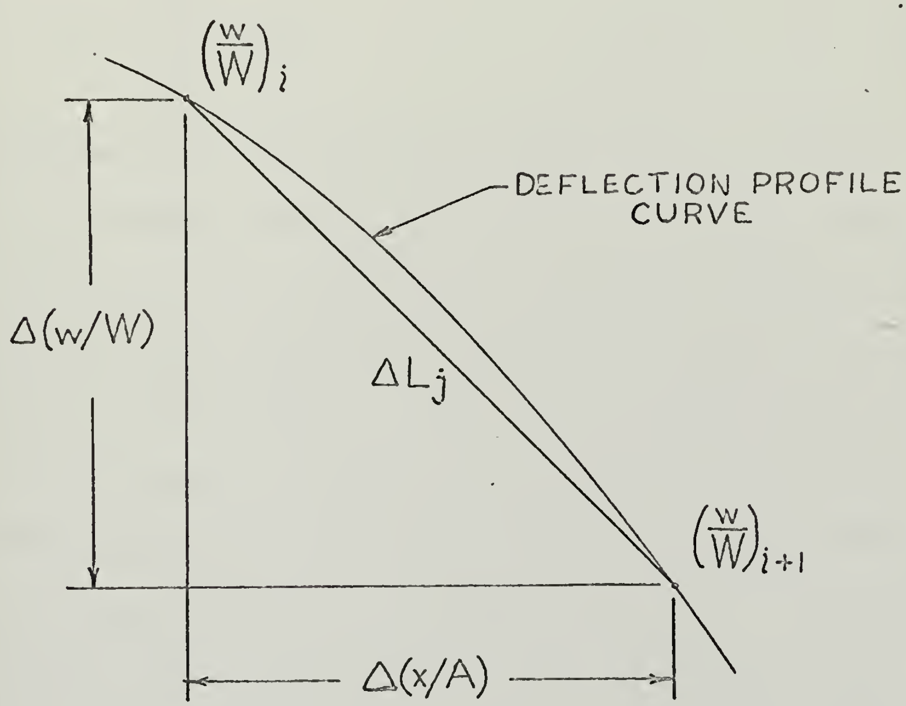


Figure D2. Element of Deflection Profile Curve

Thus,

$$\Delta L_j = \sqrt{\left[\Delta(x/A)\right]^2 + \left[\Delta(w/W)\quad(W/A)\right]^2} \tag{D3}$$

where

$$\Delta(w/W) = \left(\frac{w}{W}\right)_i - \left(\frac{w}{W}\right)_{i+1}$$

and

$$\Delta(x/A) = \left(\frac{x}{A}\right)_i - \left(\frac{x}{A}\right)_{i+1}$$

Then, the actual length of the deflection profile will be:

$$L = \sum_{j=1}^{J} \Delta L_j \tag{D4}$$

where J equals the number of divisions of the x/A axis. With this value of L, the average transverse strain can be found from equation (D2).

For the case of a square plate, the average strain in the y direction will equal the strain along the x axis. If $B/A > 1.0$, then the two strains will be different. The transverse strain will be the same as found from equation (D2). The longitudinal strain can be found using a similar relation, but the length of the curve must be adjusted to reflect the longer dimension of the plate. This can be accomplished simply by dividing the W/A term by B/A so that the parameter W/B is used in equation (D3) in place of W/A. Then, if L_j' is the length of the arc along the y-axis, the average longitudinal strain is:

$$\epsilon_y = \frac{L_j' - B}{B} \tag{D5}$$

Finally, the principle strain at the mid-point of the plate can be determined from equation (C10) of Appendix C, which is repeated

here for convenience.

$$\epsilon_1 = \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2} \tag{D6}$$

This is the value of strain that is presented in Tables D1 to D5 and Figure D3 for various values of B/A, C/A, and W/A. The strain is referred to as the average transverse strain in the tables because only deflections along the x-axis are computed in the analysis.

Also, it should be noted that Poisson's Ratio is equal to 0.5 for the calculations. This is the value that is commonly used for strains in the plastic range. Both ν = 0.3 and 0.5 were used in the calculations to test the validity of the membrane formula and ν = 0.5 generally gave the best results.

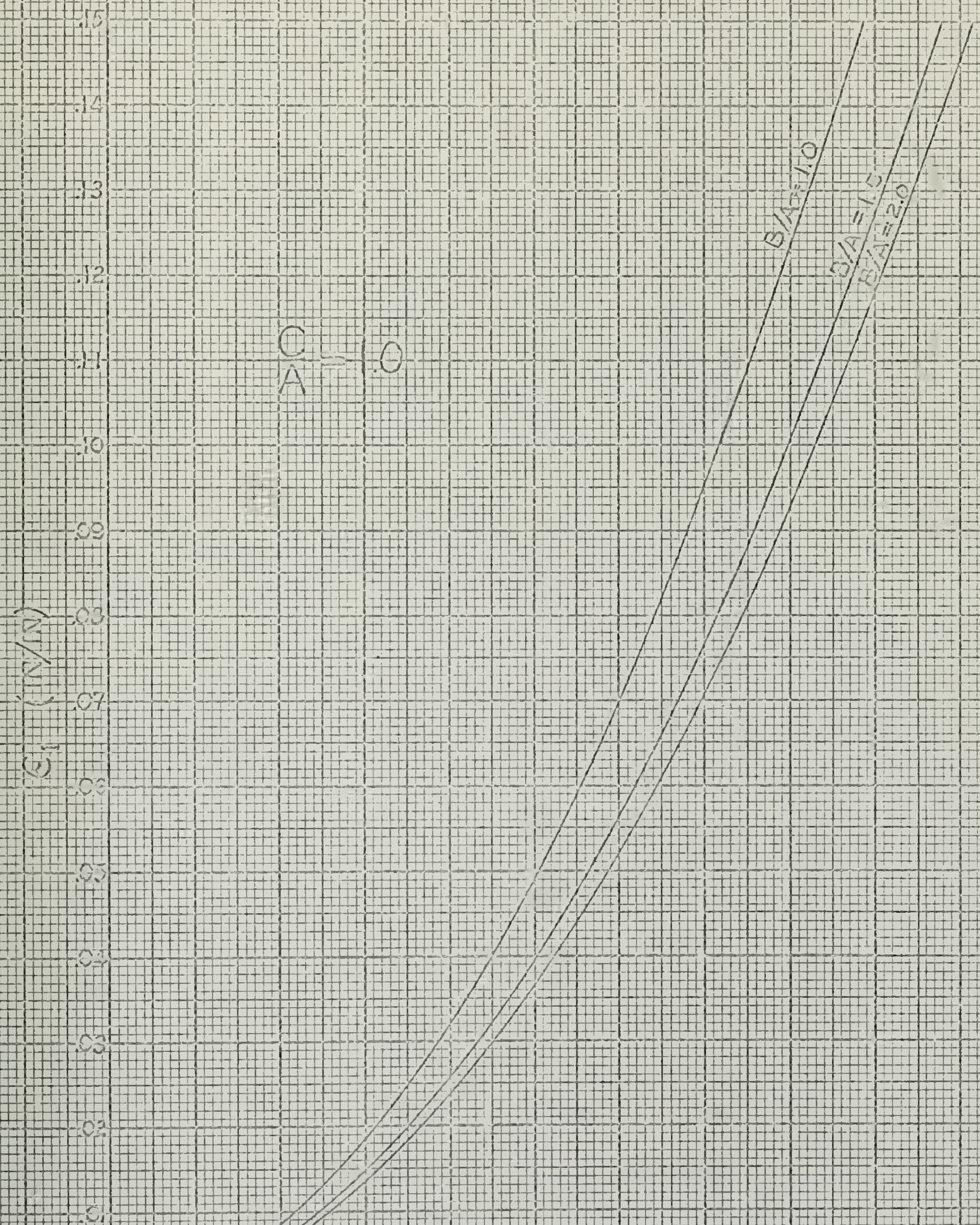


Figure D3. Average Transverse Strain at the Mid-Point of the Plate (See Equation [illegible])

TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN

B/A= 1.000 POISSON S RATIO= 0.500

RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.

C/A	W(MAX)	X/A .50	.40	.30	.20	.10
0.020	0.00035	1.0000	0.9156	0.7268	0.5034	0.2546
0.040	0.00141	1.0000	0.9162	0.7278	0.5040	0.2549
0.060	0.00316	1.0000	0.9171	0.7293	0.5049	0.2556
0.080	0.00559	1.0000	0.9182	0.7312	0.5061	0.2564
0.100	0.00868	1.0000	0.9196	0.7336	0.5077	0.2574
0.200	0.03325	1.0000	0.9281	0.7492	0.5190	0.2643
0.400	0.11500	1.0000	0.9431	0.7828	0.5508	0.2828
0.600	0.20991	1.0000	0.9505	0.8058	0.5807	0.3021
0.800	0.28338	1.0000	0.9537	0.8177	0.5999	0.3177
1.000	0.31078	1.0000	0.9549	0.8213	0.6063	0.3239

AVERAGE TRANSVERSE STRAIN FOR GIVEN MAX DEFL.

C/A \ W/A	0.025	0.050	0.075	0.100	0.150	0.200
0.020	0.00276	0.01102	0.02470	0.04365	0.09658	0.16832
0.040	0.00276	0.01103	0.02472	0.04369	0.09677	0.16847
0.060	0.00277	0.01105	0.02476	0.04376	0.09690	0.16869
0.080	0.00277	0.01107	0.02480	0.04384	0.09708	0.16899
0.100	0.00278	0.01110	0.02486	0.04395	0.09730	0.16935
0.200	0.00283	0.01128	0.02528	0.04467	0.09833	0.17186
0.400	0.00295	0.01175	0.02632	0.04647	0.10264	0.17808
0.600	0.00305	0.01217	0.02723	0.04804	0.10591	0.18335
0.800	0.00312	0.01245	0.02785	0.04911	0.10810	0.18685
1.000	0.00315	0.01255	0.02838	0.04950	0.10891	0.18813

Table D1

TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN

B/A= 1.250 POISSON S RATIO= 0.500

RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.

C/A	W(MAX)	X/A .50	.40	.30	.20	.10
0.020	0.00042	1.0000	0.9215	0.7406	0.5175	0.2637
0.040	0.00169	1.0000	0.9220	0.7414	0.5180	0.2640
0.060	0.00380	1.0000	0.9228	0.7427	0.5188	0.2645
0.080	0.00673	1.0000	0.9238	0.7444	0.5199	0.2652
0.100	0.01047	1.0000	0.9250	0.7464	0.5212	0.2661
0.200	0.04033	1.0000	0.9321	0.7596	0.5310	0.2721
0.400	0.14244	1.0000	0.9446	0.7878	0.5578	0.2878
0.600	0.26825	1.0000	0.9506	0.8065	0.5822	0.3036
0.800	0.37835	1.0000	0.9531	0.8156	0.5970	0.3156
1.000	0.44112	1.0000	0.9538	0.8179	0.6012	0.3197

AVERAGE TRANSVERSE STRAIN FOR GIVEN MAX DEFL.

C/A \ W/A	0.025	0.050	0.075	0.100	0.150	0.200
0.020	0.00246	0.00984	0.02205	0.03898	0.08639	0.15055
0.040	0.00247	0.00985	0.02206	0.03901	0.08646	0.15067
0.060	0.00247	0.00986	0.02210	0.03907	0.08658	0.15086
0.080	0.00247	0.00988	0.02214	0.03914	0.08673	0.15111
0.100	0.00248	0.00990	0.02219	0.03922	0.08691	0.15141
0.200	0.00252	0.01005	0.02252	0.03981	0.08816	0.15348
0.400	0.00261	0.01042	0.02334	0.04124	0.09118	0.15844
0.600	0.00269	0.01073	0.02402	0.04240	0.09362	0.16240
0.800	0.00274	0.01092	0.02443	0.04312	0.09512	0.16481
1.000	0.00275	0.01098	0.02456	0.04335	0.09558	0.16555

Table D2

TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN

B/A= 1.500 POISSON S RATIO= 0.500

RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.

C/A	W(MAX)	X/A .50	.40	.30	.20	.10
0.020	0.00047	1.0000	0.9242	0.7468	0.5239	0.2678
0.040	0.00187	1.0000	0.9247	0.7476	0.5244	0.2681
0.060	0.00419	1.0000	0.9254	0.7488	0.5251	0.2686
0.080	0.00741	1.0000	0.9263	0.7503	0.5261	0.2693
0.100	0.01153	1.0000	0.9274	0.7521	0.5274	0.2701
0.200	0.04456	1.0000	0.9339	0.7643	0.5364	0.2756
0.400	0.15896	1.0000	0.9453	0.7900	0.5609	0.2900
0.600	0.30376	1.0000	0.9506	0.8068	0.5829	0.3042
0.800	0.43710	1.0000	0.9528	0.8147	0.5958	0.3147
1.000	0.52354	1.0000	0.9534	0.8166	0.5991	0.3181

AVERAGE TRANSVERSE STRAIN FOR GIVEN MAX DEFL.

C/A \ W/A	0.025	0.050	0.075	0.100	0.150	0.200
0.020	0.00230	0.00917	0.02055	0.03634	0.08054	0.14035
0.040	0.00230	0.00918	0.02057	0.03637	0.08060	0.14045
0.060	0.00230	0.00919	0.02060	0.03642	0.08070	0.14062
0.080	0.00231	0.00921	0.02063	0.03648	0.08084	0.14084
0.100	0.00231	0.00923	0.02068	0.03656	0.08100	0.14111
0.200	0.00235	0.00936	0.02098	0.03708	0.08210	0.14294
0.400	0.00243	0.00968	0.02169	0.03831	0.08473	0.14725
0.600	0.00249	0.00994	0.02225	0.03929	0.08678	0.15060
0.800	0.00253	0.01009	0.02259	0.03988	0.08800	0.15255
1.000	0.00254	0.01014	0.02269	0.04004	0.08834	0.15310

Table D3

TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN

B/A= 1.750 POISSON S RATIO= 0.500

RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.

C/A	W(MAX)	X/A .50	.40	.30	.20	.10
0.020	0.00049	1.0000	0.9255	0.7498	0.5270	0.2698
0.040	0.00196	1.0000	0.9260	0.7506	0.5275	0.2701
0.060	0.00440	1.0000	0.9266	0.7517	0.5282	0.2706
0.080	0.00779	1.0000	0.9275	0.7532	0.5291	0.2712
0.100	0.01212	1.0000	0.9285	0.7542	0.5303	0.2720
0.200	0.04692	1.0000	0.9348	0.7665	0.5390	0.2772
0.400	0.16820	1.0000	0.9456	0.7910	0.5624	0.2910
0.600	0.32377	1.0000	0.9507	0.8069	0.5832	0.3045
0.800	0.47052	1.0000	0.9527	0.8143	0.5952	0.3143
1.000	0.57103	1.0000	0.9532	0.8159	0.5982	0.3173

AVERAGE TRANSVERSE STRAIN FOR GIVEN MAX DEFL.

C/A \ W/A	0.025	0.050	0.075	0.100	0.150	0.200
0.020	0.00219	0.00876	0.01963	0.03470	0.07638	0.13394
0.040	0.00220	0.00877	0.01964	0.03473	0.07694	0.13404
0.060	0.00220	0.00878	0.01967	0.03477	0.07704	0.13420
0.080	0.00220	0.00879	0.01970	0.03483	0.07716	0.13440
0.100	0.00221	0.00881	0.01974	0.03490	0.07731	0.13465
0.200	0.00224	0.00893	0.02002	0.03538	0.07833	0.13634
0.400	0.00231	0.00923	0.02067	0.03652	0.08074	0.14030
0.600	0.00237	0.00946	0.02113	0.03741	0.08260	0.14333
0.800	0.00241	0.00960	0.02148	0.03792	0.08368	0.14506
1.000	0.00242	0.00963	0.02156	0.03806	0.08396	0.14551

Table D4

TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN

B/A= 2.000   POISSON S RATIO= 0.500

RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.

C/A	W(MAX)	X/A .50	.40	.30	.20	.10
0.020	0.00050	1.0000	0.9262	0.7513	0.5285	0.2708
0.040	0.00201	1.0000	0.9266	0.7520	0.5290	0.2711
0.060	0.00451	1.0000	0.9273	0.7531	0.5297	0.2715
0.080	0.00800	1.0000	0.9281	0.7546	0.5306	0.2722
0.100	0.01244	1.0000	0.9291	0.7563	0.5318	0.2729
0.200	0.04818	1.0000	0.9352	0.7676	0.5402	0.2781
0.400	0.17315	1.0000	0.9458	0.7916	0.5631	0.2916
0.600	0.33454	1.0000	0.9507	0.8070	0.5833	0.3047
0.800	0.48864	1.0000	0.9526	0.8141	0.5949	0.3141
1.000	0.59699	1.0000	0.9531	0.8156	0.5977	0.3170

AVERAGE TRANSVERSE STRAIN FOR GIVEN MAX DEFL.

W/A C/A	0.025	0.050	0.075	0.100	0.150	0.200
0.020	0.00213	0.00848	0.01901	0.03361	0.07445	0.12966
0.040	0.00213	0.00849	0.01903	0.03363	0.07450	0.12975
0.060	0.00213	0.00850	0.01905	0.03368	0.07459	0.12990
0.080	0.00213	0.00852	0.01908	0.03373	0.07471	0.13010
0.100	0.00214	0.00853	0.01912	0.03380	0.07485	0.13033
0.200	0.00217	0.00865	0.01938	0.03426	0.07583	0.13194
0.400	0.00224	0.00893	0.02001	0.03534	0.07811	0.13569
0.600	0.00229	0.00915	0.02049	0.03617	0.07986	0.13854
0.800	0.00233	0.00928	0.02077	0.03665	0.08036	0.14015
1.000	0.00233	0.00931	0.02084	0.03678	0.08112	0.14056

Table D5

Table D6

LIST OF INPUT VARIABLES FOR COMPUTER PROGRAM FOR PLATE DEFLECTION PROFILE AND TRANSVERSE STRAIN.

Computer	Definition or Symbol in Text
Input Variables*	
BA1	First value of B/A to be used in Eqn. (D1)
C2	Increment of B/A to be added to previous value for next set of calculations
C3	Last value of B/A desired to be used in calculations
PR	Poisson's Ratio
NWA	Number of W/A values to be used in calculations
NXA	Number of x/A values to be displayed in output
WA	W/A
Output Variables**	
W(MAX)	Solution of equation (D1) for x/A = 0.5, the mid-point of the plate
x/A	For plate dimensions of Figure 2
C/A	For load definition of Figure 1
W/A	For deflection definition of Figure 3

* See statements DEFL0004 and DEFL0006 in the program listing.

** See Tables D1 - D5.

```
C  PLATE DEFLECTION PROFILE AND TRANSVERSE STRAIN AT MID-POINT              DEFL0001
      DIMENSION WA(10), ARC(10), DW(10),STN(10,10),W3(11,11),               DEFL0002
     1          YARC(10),YSTN(10,10),STRN(10,10)                            DEFL0003
      READ(5,100) BA1,C2,C3,PR,NWA,NXA                                      DEFL0004
  100 FORMAT(4F5.0,2I3)                                                     DEFL0005
      READ(5,106) (WA(I),I=1,NWA)                                           DEFL0006
  106 FORMAT(6F5.0)                                                         DEFL0007
  6   BA=BA1                                                                DEFL0008
  10  WRITE(6,103) BA,PR                                                    DEFL0009
  103 FORMAT(1H1,///12X,'TRANSVERSE DEFLECTION PROFILE AND AVG. STRAIN'     DEFL0010
     1 ,//20X,'B/A=',F6.3,3X,'POISSON S RATIO=',F6.3,//11X,                 DEFL0011
     2 'RATIO OF DEFL. COEFFIC. AT X/A TO DEFL. COEFFIC. AT MID-PT.')       DEFL0012
      WRITE(6,104)                                                          DEFL0013
  104 FORMAT(1H0,22X,'X/A .50',5X,'.40',5X,'.30',5X,'.20',5X,'.10',         DEFL0014
     1 //11X,'C/A',3X,'W(MAX)')                                             DEFL0015
      NN=1                                                                  DEFL0016
      CA=0.02                                                               DEFL0017
      DCA=0.02                                                              DEFL0018
  20  DO 21 L=1,NWA                                                         DEFL0019
      YARC(L)=0.0                                                           DEFL0020
  21  ARC(L)=0.0                                                            DEFL0021
      XA=0.5                                                                DEFL0022
      DXA=0.05                                                              DEFL0023
      MM=1                                                                  DEFL0024
  25  SUM=0.0                                                               DEFL0025
      DO 30 J=1,4                                                           DEFL0026
      M=2.0*J-1.0                                                           DEFL0027
      ALPHA=1.5708*M*BA                                                     DEFL0028
      GAMMA=.7854*M*CA                                                      DEFL0029
      BETA=3.1416*M*XA                                                      DEFL0030
      CSHA=COSH(ALPHA)                                                      DEFL0031
      F1=((-1)**((M-1)/2))*SIN(2.0*GAMMA)/M**5                              DEFL0032
      F2=COSH(ALPHA-2.0*GAMMA)                                              DEFL0033
      F3=GAMMA*SINH(ALPHA-2.0*GAMMA)                                        DEFL0034
      F4=ALPHA*SINH(2.0*GAMMA)/(2.0*CSHA)                                   DEFL0035
      S1=F1*(1.0-(F2+F3+F4)/CSHA)*SIN(BETA)                                 DEFL0036
```

```
30    SUM=SUM+S1                                                    DEFL0037
      IF (0.5-XA) 35,35,40                                          DEFL0038
35    WM=SUM                                                        DEFL0039
40    W3(MM,NN)=SUM/WM                                              DEFL0040
      W2=W3(MM,NN)                                                  DEFL0041
45    IF (0.5-XA) 56,56,57                                          DEFL0042
57    DO 55 N=1,NWA                                                 DEFL0043
      DW(N)=(W1-W2)*WA(N)                                           DEFL0044
      ARC(N)=ARC(N)+SQRT(DXA**2+DW(N)**2)                           DEFL0045
55    YARC(N)=YARC(N)+SQRT(DXA**2+(DW(N)/BA)**2)                    DEFL0046
56    W1=W2                                                         DEFL0047
      XA=XA-DXA                                                     DEFL0048
      MM=MM+1                                                       DEFL0049
      IF (XA) 26,26,25                                              DEFL0050
26    IF (W2) 50,50,27                                              DEFL0051
27    W2=0.0                                                        DEFL0052
      GO TO 57                                                      DEFL0053
50    DO 80 K=1,NWA                                                 DEFL0054
      STN(K,NN)=(ARC(K)-.50)/.50                                    DEFL0055
      YSTN(K,NN)=(YARC(K)-.50)/.50                                  DEFL0056
80    STRN(K,NN)=(STN(K,NN)+PR*YSTN(K,NN))/(1.0-PR**2)              DEFL0057
60    WRITE(6,105) CA,WM,(W3(MM,NN),MM=1,NXA,2)                     DEFL0058
105   FORMAT(1H ,9X,F5.3,F8.5,10F8.4)                               DEFL0059
      IF (CA-0.10) 62,63,61                                         DEFL0060
63    CA=0.0                                                        DEFL0061
61    DCA=0.2                                                       DEFL0062
62    CA=CA+DCA                                                     DEFL0063
      NN=NN+1                                                       DEFL0064
64    IF (CA-1.0) 20,20,65                                          DEFL0065
65    WRITE (6,109) (WA(JJ),JJ=1,NWA)                               DEFL0066
109   FORMAT(///,12X,'AVERAGE TRANSVERSE STRAIN FOR  GIVEN MAX DEFL.',   DEFL0067
     1 //11X,'W/A',6F8.3,//10X, 'C/A')                              DEFL0068
      PCA=0.02                                                      DEFL0069
      DC=0.02                                                       DEFL0070
      DO 75 I=1,10                                                  DEFL0071
      WRITE(6,108) PCA,(STRN(KK,I),KK=1,NWA)                        DEFL0072
```

```
108   FORMAT(1H0,9X,F5.3,6F8.5)                                   DEFL0073
      IF (PCA-0.10) 75,77,78                                      DEFL0074
77    PCA=0.0                                                     DEFL0075
78    DC=0.2                                                      DEFL0076
75    PCA=PCA+DC                                                  DEFL0077
      BA=BA+C2                                                    DEFL0078
      IF (BA-C3) 10,10,70                                         DEFL0079
70    CALL EXIT                                                   DEFL0080
      END                                                         DEFL0081
```

Appendix E

COMPUTER PROGRAM FOR ICE/PLATE INTERACTION

The FORTRAN IV computer program to calculate the mid-point deflection of a plate and the amount of energy absorbed at the expected point of failure is listed at the end of this appendix. The logic of the program follows the step-by-step procedure given on pages 37 to 39. The values obtained for $(C/A)_o$ from step 1 is used as input to the program. The applied force F_o and its corresponding W are determined in the first part of the program. Then the absorbed energy U is calculated from the strain relation at the mid-point of the long side of the plate. Next, the applied force is increased by a ΔF and the process is repeated to find the new value of W and U. The output format of the program is illustrated in Tables E1 to E19. The description of the input and output variables and their relation to the parameters defined on the page of Nomenclature are given in Table E20.

Similar to a hand calculation of W and U, a constitutive relation between the stress and the strain for the plate material is required for the computer program. A generalized stress-strain curve was used in the program to obtain a stress for a given total strain. The general form of the equation is:

$$\sigma = D(\lambda + \epsilon_p)^n = D(\epsilon)^n \qquad \text{(E1)}$$

where ϵ_p is the plastic strain and ϵ is the total strain of the material. With the proper choice of coefficients, this equation provides a reasonable fit to an actual tensile test stress-strain curve.

It was found that by setting λ equal to the yield strain of a given material, and using the boundary condition $\sigma = \sigma_u$ when $\epsilon = \epsilon_u$, the exponent n and strength Coefficient D can be determined. For materials such as steels which show near constant stress for the smaller values of ϵ, the value used for λ should be the strain at the point where σ ceases to be constant. For example, the stress-strain curve for mild steel exhibits a near constant stress at the yield point up to approximately $\epsilon = 0.02$ in/in. Thus $\lambda = 0.02$ should be used in Equation (E1).

Since the generalized expression is always increasing with increasing ϵ_p, the choice of ϵ_u is important to ensure a proper curve fit. It is usually difficult to determine from an actual stress-strain curve, the actual value of strain that corresponds to σ_u. The best selection of ϵ_u is at the point where the σ - ϵ curve first starts to level off. The slope of the curve may not actually reach zero until just prior to the failure strain. But the ultimate stress is usually approached to within a very small error at a strain considerably smaller than the actual ϵ_u. Therefore, in order to maintain a good fit of the curve between $\epsilon = \lambda$ and $\epsilon = \epsilon_u$, a value of ϵ_u should be used that may be less than the actual ultimate strain.

From the chosen values for λ and ϵ_u, the values for D and n are determined from:

$$n = \frac{\ln(\sigma_o / \sigma_u)}{\ln(\lambda / \epsilon_u)} \tag{E2}$$

and

$$D = \frac{\sigma_o}{\lambda^n} \tag{E3}$$

The use of the generalized stress-strain curve allowed the incorporation of the average transverse strain program discussed in Appendix D into the overall program as a subroutine. The name of this subroutine is STRAIN and its listing follows that of the main program at the end of this appendix. For an input of the variables B/A, C/A, and W/A to STRAIN, a value of ϵ_1 as given by equation (C10) is calculated and returned to the main program. Then the stress for ϵ_1 is found from the generalized stress-strain function.

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 45,2 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0,125
WIDTH= 54,00
LENGTH= 84,00

MATERIAL PROPERTIES
YIELD STRESS= 56000,0 PSI
ULT, STRESS= 71200,0 PSI
YIELD STRAIN=0,0200 IN/IN
ULT, STRAIN=0,1400 IN/IN
POISSON S RATIO=0,500

				AT MID-PT OF LONG SIDE		
F (LBS)	W (IN)	S (LB/IN)	C/A	STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
205027,1	1,9068	7000,0	1,000	0,0230	56967,5	1309,0
410027,1	3,8133	7000,0	1,000	0,0460	62054,5	2851,6
615027,1	5,2839	7577,6	1,000	0,0661	64903,2	4290,8
820027,1	6,6620	8013,4	1,000	0,0857	67012,7	5741,0
1025027,1	7,9763	8366,2	1,000	0,1048	68701,9	7201,2

Table E1. Plate 1B of Reference E3

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 48.9 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.182
WIDTH= 54.00
LENGTH= 84.00

MATERIAL PROPERTIES
YIELD STRESS= 41800.0 PSI
ULT. STRESS= 65500.0 PSI
YIELD STRAIN=0.0175 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
221810.3	1.8981	7607.6	1.000	0.0333	47631.6	1586.3
443810.3	3.7318	7742.2	1.000	0.0660	54727.4	3612.8
665810.3	4.9842	8696.5	1.000	0.0933	58702.7	5474.4
887810.3	6.1231	9439.3	1.000	0.1194	61725.4	7371.9
1109810.0	7.1896	10049.3	1.000	0.1449	64197.4	9303.7

Table E2. Plate 2A of Reference E3

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 241.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.244
WIDTH= 13.50
LENGTH= 21.00

MATERIAL PROPERTIES
YIELD STRESS= 56000.0 PSI
ULT. STRESS= 70700.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1400 IN/IN
POISSON S RATIO=0.500

				AT MID-PT OF LONG SIDE		
F (LBS)	W (IN)	S (LB/IN)	C/A	STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
68323.5	0.3255	13664.0	1.000	0.0306	53933.2	1805.0
136523.5	0.6505	13664.0	1.000	0.0612	64028.4	3918.7
204723.5	0.9714	13720.1	1.000	0.0916	67194.8	6153.3
272923.5	1.2257	14496.1	1.000	0.1186	69307.7	8218.7
341123.5	1.4683	15125.1	1.000	0.1450	70997.8	10294.8

Table E3. Plate P2 of Reference E2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 190.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.119
WIDTH= 13.50
LENGTH= 21.00

MATERIAL PROPERTIES
YIELD STRESS= 56000.0 PSI
ULT. STRESS= 70700.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1400 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
53865.0	0.5262	6664.0	1.000	0.0241	57278.4	1383.1
107665.0	1.0332	6783.9	1.000	0.0478	62161.7	2971.8
161465.0	1.4329	7336.0	1.000	0.0688	64935.4	4469.3
215265.0	1.8091	7746.4	1.000	0.0893	66990.1	5980.3
269065.0	2.1707	8069.4	1.000	0.1094	68638.4	7506.0

Table E4. Plate P3 of Reference E2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 120.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.105
WIDTH= 13.50
LENGTH= 21.00

MATERIAL PROPERTIES
YIELD STRESS= 32000.0 PSI
ULT. STRESS= 46100.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
34020.0	0.6592	3360.0	1.000	0.0267	33662.7	898.4
68020.0	1.2120	3653.6	1.000	0.0511	37724.1	1926.3
102020.0	1.6380	4054.8	1.000	0.0727	40136.2	2917.2
136020.0	2.0307	4360.6	1.000	0.0936	41957.1	3926.3
170020.0	2.4000	4611.8	1.000	0.1140	43434.5	4950.2

Table E5. Plate P4 of Reference E2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 87.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.068
WIDTH= 13.50
LENGTH= 21.00

MATERIAL PROPERTIES
YIELD STRESS= 32000.0 PSI
ULT. STRESS= 41800.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

				AT MID-PT OF LONG SIDE		
F (LBS)	W (IN)	S (LB/IN)	C/A	STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
24664.5	0.7379	2176.0	1.000	0.0193	31864.3	616.6
49284.5	1.3536	2370.4	1.000	0.0370	34627.5	1280.0
73904.5	1.8709	2571.6	1.000	0.0532	36285.5	1930.3
98524.5	2.3545	2724.2	1.000	0.0690	37515.4	2586.8
123144.5	2.8160	2846.9	1.000	0.0844	38502.4	3249.6

Table E6. Plate P6 of Reference E2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 184.2 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.119
WIDTH= 13.50
LENGTH= 21.00

MATERIAL PROPERTIES
YIELD STRESS= 42000.0 PSI
ULT. STRESS= 61000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
52220.7	0.6802	4998.0	1.000	0.0312	45492.8	1420.0
104420.7	1.2409	5478.4	1.000	0.0595	51074.1	3037.9
156620.7	1.6747	6088.3	1.000	0.0846	54410.7	4604.6
208820.7	2.0726	6559.1	1.000	0.1089	56928.9	6198.9
261020.7	2.4442	6952.4	1.000	0.1325	58969.9	7813.6

Table E7. Plate P8 of Reference E2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 100.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.188
WIDTH= 13.00
LENGTH= 36.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

				AT MID-PT OF LONG SIDE		
F (LBS)	W (IN)	S (LB/IN)	C/A	STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
46800.0	0.2843	7238.0	1.000	0.0205	38691.0	792.7
140300.0	0.8524	7238.0	1.000	0.0614	48472.2	2977.2
233800.0	1.2766	8053.7	1.000	0.0968	53216.3	5151.0
327300.0	1.6229	8869.0	1.000	0.1292	56462.5	7292.5
420800.0	1.9440	9519.0	1.000	0.1605	59041.5	9479.1

Table E8. Plate no. 2 of Reference E1

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 100.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.188
WIDTH= 13.00
LENGTH= 19.25

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
25025.0	0.2365	7238.0	1.000	0.0178	37590.4	669.2
58825.0	0.5558	7238.0	1.000	0.0418	44799.8	1874.6
92625.0	0.8752	7238.0	1.000	0.0659	49175.8	3240.1
126425.0	1.1059	7818.5	1.000	0.0864	51983.4	4488.8
160225.0	1.3089	8371.6	1.000	0.1057	54186.4	5727.4
194025.0	1.5001	8845.6	1.000	0.1246	56044.7	6981.3
227825.0	1.6822	9262.3	1.000	0.1431	57660.8	8249.5
261625.0	1.8569	9635.4	1.000	0.1613	59096.2	9510.8
295425.0	2.0256	9974.1	1.000	0.1792	60391.2	10824.3

Table E9. Plate no. 3 of Reference E1

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 100.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.188
WIDTH= 13.00
LENGTH= 13.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
16900.0	0.1717	7238.0	1.000	0.0142	35870.3	508.3
50700.0	0.5151	7238.0	1.000	0.0425	44944.9	1910.5
84500.0	0.8482	7325.9	1.000	0.0704	49848.0	3508.9
118300.0	1.0764	8081.4	1.000	0.0936	52855.1	4949.2
152100.0	1.2865	8630.0	1.000	0.1162	55247.1	6417.9
185900.0	1.4866	9155.1	1.000	0.1381	57241.9	7903.8
219700.0	1.6750	9645.2	1.000	0.1596	58966.2	9408.3
253500.0	1.8555	10046.3	1.000	0.1807	60491.2	10930.0
287300.0	2.0296	10409.0	1.000	0.2015	61862.6	12467.5

Table E10. Plate no. 5 of Reference E1

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 100,0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0,250
WIDTH= 13,00
LENGTH= 26,00

MATERIAL PROPERTIES
YIELD STRESS= 66000,0 PSI
ULT. STRESS= 83000.0 PSI
YIELD STRAIN=0,0200 IN/IN
ULT. STRAIN=0,1200 IN/IN
POISSON S RATIO=0,500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
33800,0	0,1167	16500,0	1,000	0,0114	61412,9	699,4
101300,0	0,3497	16500,0	1,000	0,0341	70669,7	2412,0
168800,0	0,5827	16500,0	1,000	0,0569	75439,6	4290,5
236300,0	0,8157	16500,0	1,000	0,0796	78756,4	6270,2
303800,0	1,0289	16817,1	1,000	0,1013	81223,2	8229,8
371300,0	1,2075	17513,8	1,000	0,1212	83107,2	10074,1

Table E11. Plate no. 4 of Reference E1

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.750
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
42336.0	0.8001	28875.0	0.350	0.0757	50595.7	3829.4
142335.9	1.2013	28875.0	0.642	0.1388	57301.5	7952.3
242335.9	1.3350	28875.0	0.837	0.1811	60518.4	10958.8
342335.9	1.3636	28875.0	0.995	0.2152	62702.8	13495.2
442335.9	1.5424	28875.0	1.000	0.2446	64374.1	15749.1
542335.9	1.7079	28875.0	1.000	0.2709	65735.0	17807.3
642335.9	1.8308	29316.0	1.000	0.2924	66775.7	19527.6

Table E12. Arbitrary Plate no. 1

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.000
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
104543.9	0.8259	38500.0	0.550	0.1189	55515.0	6602.8
204543.9	0.9756	38500.0	0.769	0.1664	59474.3	9894.4
304543.9	1.0211	38500.0	0.939	0.2030	61954.5	12576.6
404543.9	1.1063	38500.0	1.000	0.2340	63786.7	14923.8
504543.9	1.2355	38500.0	1.000	0.2613	65249.5	17048.8
604543.9	1.3524	38500.0	1.000	0.2860	66471.7	19011.6

Table E13. Arbitrary Plate no. 2

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.250
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
279935.7	0.8129	48125.0	0.900	0.1946	61421.0	11954.0
379935.7	0.8577	48125.0	1.000	0.2267	63377.1	14369.9
479935.7	0.9640	48125.0	1.000	0.2548	64915.5	16542.7
579935.7	1.0597	48125.0	1.000	0.2801	66188.8	18541.4
679935.7	1.1474	48125.0	1.000	0.3033	67278.5	20406.9

Table E14. Arbitrary Plate no. 3

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.500
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 38500.0 PSI
ULT. STRESS= 59000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1600 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
691200.0	0.6817	57750.0	1.000	0.2162	62764.0	13572.6
791200.0	1.0314	57750.0	1.000	0.3272	68333.2	22358.5
891200.0	1.0947	57750.0	1.000	0.3473	69173.1	24021.1
991200.0	1.1545	57750.0	1.000	0.3662	69932.4	25611.0

Table E15. Arbitrary Plate no. 4

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=0.750
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 66000.0 PSI
ULT. STRESS= 83000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1200 IN/IN
POISSON S RATIO=0.500

				AT MID-PT OF LONG SIDE		
F (LBS)	W (IN)	S (LB/IN)	C/A	STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
104543.9	0.6423	49500.0	0.550	0.0694	77382.3	5368.8
254543.9	0.7833	49500.0	0.858	0.1083	81914.0	8867.9
404543.9	0.8605	49500.0	1.000	0.1365	84377.4	11515.8
554543.9	1.0074	49500.0	1.000	0.1598	86096.6	13757.4
704543.9	1.1355	49500.0	1.000	0.1801	87425.0	15746.1
854543.9	1.2506	49500.0	1.000	0.1984	88510.9	17556.9

Table E16. Arbitrary Plate no. 5

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.000
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 66000.0 PSI
ULT. STRESS= 83000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1200 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
221183.9	0.5765	66000.0	0.800	0.1009	81181.3	8192.5
371183.9	0.6182	66000.0	1.000	0.1307	83914.2	10970.2
521183.9	0.7325	66000.0	1.000	0.1549	85755.7	13284.4
671183.9	0.8313	66000.0	1.000	0.1758	87154.2	15321.2
821183.9	0.9195	66000.0	1.000	0.1944	88285.8	17167.0
971183.9	0.9999	66000.0	1.000	0.2115	89238.2	18870.6

Table E17. Arbitrary Plate no. 6

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.250
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 66000.0 PSI
ULT. STRESS= 83000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1200 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
691200.0	0.4772	82500.0	1.000	0.1261	83531.9	10537.1
891200.0	0.7663	82500.0	1.000	0.2026	88749.0	17977.7
1091200.0	0.8479	82500.0	1.000	0.2241	89905.7	20152.2
1291200.0	0.9224	82500.0	1.000	0.2438	90878.6	22158.6
1491200.0	1.0295	82500.0	1.000	0.2721	92164.9	25082.4

Table E18. Arbitrary Plate no. 7

ICE/PLATE INTERACTION

ICE CRUSHING PRESSURE= 600.0 PSI

PLATE DIMENSIONS (INCHES)
THICKNESS=1.500
WIDTH= 24.00
LENGTH= 48.00

MATERIAL PROPERTIES
YIELD STRESS= 66000.0 PSI
ULT. STRESS= 83000.0 PSI
YIELD STRAIN=0.0200 IN/IN
ULT. STRAIN=0.1200 IN/IN
POISSON S RATIO=0.500

F (LBS)	W (IN)	S (LB/IN)	C/A	AT MID-PT OF LONG SIDE STRAIN (IN/IN)	STRESS (PSI)	U (IN-LB)
691200.0	0.3977	99000.0	1.000	0.1261	83531.9	10537.1
891200.0	0.6386	99000.0	1.000	0.2026	88749.0	17977.7
1091200.0	0.7066	99000.0	1.000	0.2241	89905.7	20152.2
1291200.0	0.7686	99000.0	1.000	0.2438	90878.6	22158.6
1491200.0	0.8579	99000.0	1.000	0.2721	92164.9	25082.4

Table E19. Arbitrary Plate no. 8

Table E20

LIST OF INPUT VARIABLES FOR THE COMPUTER PROGRAM

FOR PLATE DEFLECTION/LOAD RELATION

Computer Input Variables*	Definition or Symbol in Text
YIELD	σ_o
ULT	σ_u
EY	λ in equation (E1)
EF	ϵ_u
PR	Poisson's Ratio
DF	ΔF
FLIM	Last value of F to be used in the calculations
H	H
A	A
B	B
CA	$(C/A)_o$
PC	P_c
Q	k
QD	k_d

Output Variables (See Tables E1 to E19)

All of the variables displayed in the output are self explanatory or of the same notation as used in the text. The STRAIN at the mid-point of the long side is equal to ϵ_t in equation (19) of the text. The STRESS at the same point is equal to σ_t.

* See statements FAIL0003 and FAIL0005 in the program listing.

```
C     PLATE DEFLECTION/LOAD RELATION                                    FAIL0001
      SIGMAF(ST)=SCO*(ST**PWR)                                          FAIL0002
  370 READ(5,340) YIELD,ULT,EY,EF,PR,DF,FLIM                            FAIL0003
      IF (YIELD) 500,500,375                                            FAIL0004
  375 READ(5,340) H,A,B,CA,PC,Q,QD                                      FAIL0005
  340 FORMAT(8F10.0)                                                    FAIL0006
      WRITE(6,305) PC,H,A,B                                             FAIL0007
  305 FORMAT(1H1,///30X,'ICE/PLATE INTERACTION',//20X,                  FAIL0008
     1 'ICE CRUSHING PRESSURE=',F7.1,' PSI',//20X,                      FAIL0009
     2 'PLATE DIMENSIONS (INCHES)',/26X,'THICKNESS=',F5.3,/26X,         FAIL0010
     3 'WIDTH=',F8.2,/26X,'LENGTH=',F7.2)                               FAIL0011
      WRITE(6,315) YIELD,ULT,EY,EF,PR                                   FAIL0012
  315 FORMAT(//20X, 'MATERIAL PROPERTIES',/26X,'YIELD STRESS=',F8.1,    FAIL0013
     1 ' PSI',/26X,'ULT. STRESS=',F9.1,' PSI',/26X,'YIELD STRAIN=',     FAIL0014
     2 F6.4,' IN/IN',/26X,'ULT.  STRAIN=',  F6.4,' IN/IN',/26X,         FAIL0015
     3 'POISSON S RATIO=',F5.3)                                         FAIL0016
      WRITE(6,379)                                                      FAIL0017
  379 FORMAT(1HC,45X,'AT MID-PT OF LONG SIDE',/13X,'F',8X,'W',          FAIL0018
     1 8X,'S',6X,'C/A',3X,'STRAIN  STRESS',7X,'U',/11X,                 FAIL0019
     2 '(LBS)',5X,'(IN)',3X,'(LB/IN)', 8X,'(IN/IN)   (PSI)',4X,         FAIL0020
     3 '(IN-LB)')                                                       FAIL0021
      PI3=3.1416**3                                                     FAIL0022
      PWR=ALOG(YIELD/ULT)/ALOG(EY/EF)                                   FAIL0023
      SCO=YIELD/(EY**PWR)                                               FAIL0024
      BA=B/A                                                            FAIL0025
      C=CA*A                                                            FAIL0026
      IF (CA-1.0) 105,110,110                                           FAIL0027
  110 C1=B                                                              FAIL0028
      GO TO 115                                                         FAIL0029
  105 C1=C                                                              FAIL0030
  115 F=PC*C*C1                                                         FAIL0031
      PL=PC                                                             FAIL0032
      S=YIELD*H                                                         FAIL0033
  360 ITDEX=0.0                                                         FAIL0034
      TOLER=1.0                                                         FAIL0035
  300 ZM=(16.0*((C/2.0)**2)*Q*PL)/(PI3*S)                               FAIL0036
```

```
      DZM=(PL*C*QD)/(2.0*S)                                          FAIL0037
      W=ZM+0.5*(A-C)*DZM                                             FAIL0038
      IF (TCLER) 310,310,390                                         FAIL0039
  390 WA=W/A                                                         FAIL0040
      CALL STRAIN(BA,CA,WA,PR,STRN)                                  FAIL0041
      IF (STRN-EY) 410,410,405                                       FAIL0042
  410 SN=YIELD*H                                                     FAIL0043
      GO TC 415                                                      FAIL0044
  405 SN=H*SIGMAF(STRN)                                              FAIL0045
  415 TCLER=ABS(SN-S)                                                FAIL0046
      IF (TOLER-25.0) 395,395,290                                    FAIL0047
  395 TOLER=0.0                                                      FAIL0048
      S=S+(SN-S)/2.0                                                 FAIL0049
      GO TO 300                                                      FAIL0050
  290 S=SN                                                           FAIL0051
      ITDEX=ITDEX+1                                                  FAIL0052
      IF (ITDEX-5) 300,300,400                                       FAIL0053
  310 EX=(H/2.0)*DZM*((S/H)/(1.156*YIELD)+1.0)/(1.0-PR**2)           FAIL0054
      STRS=SIGMAF(EX)                                                FAIL0055
      U=EX*STRS                                                      FAIL0056
      WRITE(6,380) F,W,S,CA,EX,STRS,U                                FAIL0057
  380 FORMAT(1H0,8X,F9.1,F8.4,F9.1,F7.3,F8.4,2F9.1)                  FAIL0058
  350 F=F+DF                                                         FAIL0059
      IF (F-FLIM) 345,345,370                                        FAIL0060
  345 IF (CA-1.0) 325,130,130                                        FAIL0061
  325 C=SQRT(F/PC)                                                   FAIL0062
      IF (C-A) 140,125,130                                           FAIL0063
  140 CA=C/A                                                         FAIL0064
      PL=PC                                                          FAIL0065
      GO TO 360                                                      FAIL0066
  130 C1=SQRT(F/PC)                                                  FAIL0067
      IF (B-C1) 120,126,126                                          FAIL0068
  120 C1=B                                                           FAIL0069
  126 PL=F/(A*C1)                                                    FAIL0070
      GO TO 145                                                      FAIL0071
  125 PL=PC                                                          FAIL0072
```

```
145  CA=1.0                                                                FAIL0073
     C=A                                                                   FAIL0074
     GO TO 360                                                             FAIL0075
400  WRITE(6,349) SN,S,W                                                   FAIL0076
349  FORMAT(1H0,15X,'ITERATION INDEX EXCEEDED',/20X,'SN=',F10.2,3X,        FAIL0077
    1 'S=',F10.2,3X,'W=',F10.4)                                            FAIL0078
     GO TO 350                                                             FAIL0079
500. CALL EXIT                                                             FAIL0080
     END                                                                   FAIL0081
```

```
      SUBROUTINE STRAIN (BA,CA,WA,PR,STRN)
C   PLATE DEFLECTICN PROFILE ANC TRANSVERSE STRAIN AT MID-POINT
      ARC=0.0
      YARC=0.0
      XA=0.5
      DXA=0.05
 25   SUM=0.0
      DO 30 J=1,4
      M=2.0*J-1.0
      ALPHA=1.5708*M*BA
      GAMMA=.7854*M*CA
      BETA=3.1416*M*XA
      CSHA=CCSH(ALPHA)
      F1=((-1)**((M-1)/2))*SIN(2.0*GAMMA)/M**5
      F2=COSH(ALPHA-2.0*GAMMA)
      F3=GAMMA*SINH(ALPHA-2.0*GAMMA)
      F4=ALPHA*SINH(2.0*GAMMA)/(2.0*CSHA)
      S1=F1*(1.0-(F2+F3+F4)/CSHA)*SIN(BETA)
 30   SUM=SUM+S1
      IF (0.5-XA) 35,35,40
 35   WM=SUM
 40   W2=SUM/WM
 45   IF (C.5-XA) 56,56,57
 57   DW=(W1-W2)*WA
      ARC =ARC+SQRT(DXA**2+DW**2)
 55   YARC=YARC+SQRT(DXA**2+(DW/BA)**2)
 56   W1=W2
      XA=XA-DXA
      IF (XA) 26,26,25
 26   IF (W2) 50,5C,27
 27   W2=0.0
      GO TC 57
 50   STN=(ARC-.50)/.50
      YSTN=(YARC-.5C)/.50
      STRN=(STN+PR*YSTN)/(1.0-PR**2)
      RETURN
```

STRN0037

END

BIBLIOGRAPHY

The extent of use made of each of the following references is indicated by listing them in approximate descending order within each subject heading.

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